OPTIMIZATION

Concepts and Methodology for

DESIGN ENGINEERING

Design Engineering Monography Series, No: 01

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The best is the enemy of the good.
Voltaire

Scientists study the world as it is;

Engineers create the world that has never been.

Theodore von Karman

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PREFACE

This book is a collection of problems and their related optimum design strategies and methodologies for optimum design cases at the level of undergraduate engineering design courses. Most of the design cases and the problems were selected from examination questions of the senior level engineering design courses and home/term studies assigned to the students during 1974-2002 terms in the department of mechanical engineering of the Middle East Technical University. I have reviewed the full contents and re-written some of the text to improve the contents and make it suitable for senior students in any engineering program.

The first part of the book reviews the structure of the mathematical models of general-purpose optimization problems, their structural formulation and related methodologies and procedural steps towards a feasible design product. Special emphasis is given to engineering design cases and engineering-related problems, without any mention and/or any hint about future design-based solution approaches. This is a necessary requirement, because the final product details are not known in the early stages of any design procedure. The physical product at this stage is not created yet, hence it is a usual practice to apply functional optimization techniques after the creative phase of the design activity[1]. However, this is not a necessary step for experienced designers. The optimization methodologies in creative phase must be carried in functional domain to develop and reach the best parameters for better functional performance. This fact is not much emphasized in the book, because the book is designed for undergraduate students and optimization on functional domain may require higher level expertise on optimization techniques and an engineering or scientific expertise on a narrow field.

The second part of the book is about methodologies of optimization. The contents in this chapter are focused on graphical representation and mathematical models.

The third part of the book is about the solutions procedures of the problems in the second part.

I hope that the presented approach and basic phenomena for the design cases will help design engineers to develop better functional designs and feasible products.

I would like to thank all my students for their best efforts to solve these problems during the limited time in the examinations. I would like to thank Miss Asuman Eripek also for her meticulous typing of the manuscript in 1984 during her work period in METU.

I hope that the presented approaches in the book; the basic phenomena for the design cases and solution methodologies would help design engineers to develop better functional design products in their professional life.

Abdülkadir Erden
November 06, 2024, Ankara

[1] You may refer to the authors 'Design Engineering' books for more details on the functional design.

PART I

PROBLEMS of OPTIMIZATION

Engineering is a series of human activities to create first, and then solve problem bundles…

CHAPTER I
CONCEPTS and PHILOSOPHY of OPTIMIZATION

I.1 INTRODUCTION

If you would question your colleagues for a verbal definition of "*Engineering Design*", you should expect differing definitions from each person who answers your question. This is because every person has a unique work and is interested and specialized in the narrow field of his/her work area. Although the answers normally differing verbally, it is possible to find many common features among the given verbal definitions. One of these common basic features of engineering design is that some **DECISIONS** must be made during the design procedure. Specifying the geometry of a functional product, machine and machine elements, selection of material, all are typical activities of mechanical engineering design.

The design engineers have the all the responsibility for acceptable and successful functioning of the product; however, they do not have much freedom in their design work. Main issue to satisfy the future product (***designet)*** performance, is to perform a well-defined function, or more generally it should serve for the predetermined purposes and satisfy the pre-defined requirements. A car engine creates power by using some kind of fuel or energy storage element. Then a shaft transmits produced power to the drive mechanism or wheels of the car. Efficiency, high performance, high comfort and safety are some of the common requirements in this respect. As an alternative requirement, a toy car for children is preferred to be attractive and safe only. These humanious statements are user-initiated expectations and requests. They are declared explicitly mostly by the future users explicitly. More statements are added by the social and commercial entities, units and organizations. All statements make a basis for future decisions on the designers' side. All statements related to requests and expectations are called **REQUIREMENTS** of the design. Any product feature and design/engineering activity during the design procedure cannot be accepted as satisfying and a valid decision, unless these requirements are satisfied fully by the future design product.

In addition to the requirements, **LIMITATIONS** of a design activity and entity must be considered in any design process and satisfied fully by the designer. As it is expected that limitations of a design activity limit the application in several ways, mainly by the laws of nature and engineering science. In addition, human originated biological and social expectations create desirable features of a design product. The car engine case, given above, must develop a certain amount of power, but at the same time, it should be small and light so that it can be placed properly within the car body, and should not limit the passenger comfort. A thermally perfect engine, weighing several tons, is not a good solution (limit is not satisfied) for the present-day cars. The limitation (or one of the limitations) for the toy in the above example, is that the toy should not be dangerous and hazardous to the children. Thus, Power and Weight, or Attractiveness and danger are equally important for decisions of the designer. It is certain that the requirements and limitations (therefore decisions) must be based on acceptable, valid and satisfying available scientific and technological principles.

"*Decision*" is not a first step, nor it is a last step in the design process. It always comes after a mental activity, called **CREATIVITY**. Creativity is usually and intimately based on accumulated engineering experience and on some previous works which were found to be unsuccessful or incomplete at the recent stages of engineering development. Design creativity is the most fruitful step of the design process since it sprouts mainly in the designer's imagination with an engineering and scientific background.

After the design is completed and if it is found to be successful, the next activity is **MANUFACTURING**. It is the production of the physical item in required quantities.

Any design procedure usually results in more than one solution and all these solutions may satisfy the Requirements and Limitations. Theoretically, the number of these solutions is infinite. Each of these solutions (designs) is an alternative design product and is called a ***feasible design***. At this stage, the designer faces the problem of selecting the best of these alternatives (Feasible Designs). The best design is named ***optimum design***, -best in a predetermined way- of these feasible designs, and it is determined by using a concept/criterion. The mathematical procedure to obtain the specific solution (Optimum design product) is called **OPTIMIZATION**. The schematic block diagram in *Figure I.1* illustrates the place of optimization in a design process and other related activities.

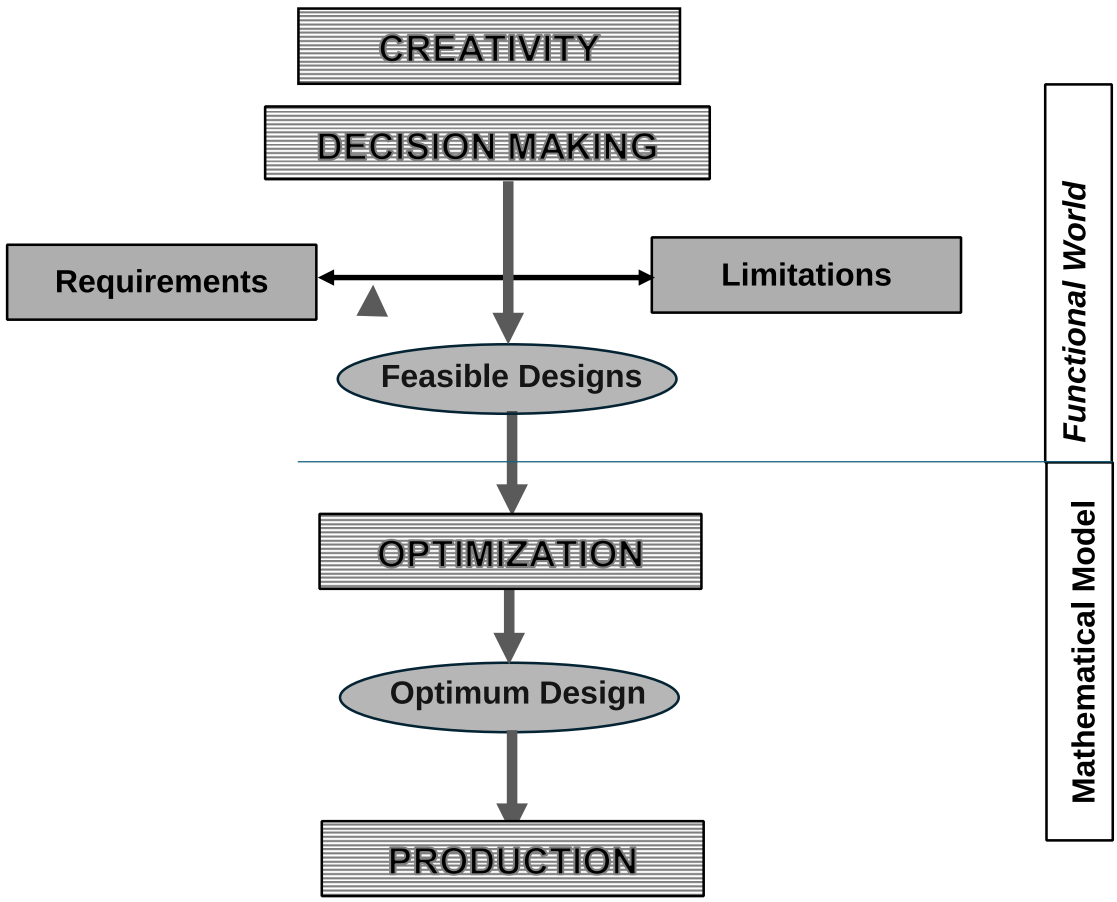


Figure I.1 Structure of Engineering Design Procedure

Flow of activities stepwise in Figure I.1 is known as design **MODELLING**. It is the main engine4ering activity between Creativity and Manufacturing to lead designers to the final product. The MODEL can be physical or mathematical depending on the specific design problem, designer's background and available facilities. Usually experimental approaches (physical modelling) are expensive and time consuming since it requires production of a physical prototype or model which simulates the physical system under working conditions that are highly like the actual environment. Although results are more reliable in physical modeling, it is impractical and costly in most cases. Mathematical modelling approaches are applied satisfactorily in these cases/problems, if the results are sufficient to obtain valid and acceptable conclusions. In the case of optimization, mathematical modelling is the only applicable method since the production of several physical models, slightly differing from each other is not a feasible and practical approach at all. Conclusively, optimization procedures follow mathematical modelling, which is mainly a paperwork with sophisticated conceptual models. In some unique cases, it may be possible to find an exceptional application with reasonable and affordable work after experimental tests. A typical case may be spacecraft design, a case where failure of the design product endangers life of the astronauts, and high costs. Hence the probability of failure should be eliminated fully.

Our aim in this text is to investigate concepts, methods, and applications of optimization by mathematical modelling. Therefore, it is necessary to discuss concepts of the previous paragraphs, Decision, Requirements and Limitations in mathematical models. Before working on these issues, some important concepts of optimization procedure should be discussed briefly.

I.2 CONCEPTS IN OPTIMIZATION

Widespread applications of optimization procedures in conventional engineering design are rather new, because of the limitations on computational requirements. It is a frequent and usual practice to select one of the the machine elements from an available list. This effortless step optimizes the designer's *Time* and *Effort* (also Project *Cost*). Is well known that no engineering project can be completed theoretically by proper and detailed applications of all the related scientific theories within a practical contract schedule. The probability of error is balanced against the cost of improvement time and known errors are within the concept of "*Safety Factor*". Good judgement on the part of the designer is essential in making reasonable "*Approximations*" in the design and arriving at acceptable compromises among the alternatives. Accuracy of design should not be sacrificed for simplicity of the computations, but they are both desirable. Thus, a successful designer is considered as the designer who makes valid approximations in design computations without loss in accuracy.

As approximations are inevitable in engineering design problems, assumptions are also necessary for design solutions within the limited time given to the designer. Usually, physical systems can be modelled mathematically only after several assumptions are made. These simplify the mathematics involved in modelling the actual physical system and make possible a practical solution.

In working on engineering design, designers cannot isolate the problem from the surrounding conditions. Besides the effects of the neighborhood machines, there are some limitations which have become quite important in recent years. We cannot use as much energy as we like, and the supply of fresh water and even fresh air are limited. The environmental pollution problem should be considered anywhere in the world with a engineering design product. Thus "the Optimum Solution" and "the Optimal Solution" may not be same for most of the problems. We may develop an ideally optimum design, but it is usually never possible to produce it.

When a designer mentions Optimization, he means ***Maximization*** or ***Minimization*** of a certain **CRITERION** for a **SYSTEM**. "System", in general, defines the boundaries of engineer's or designer's concern. It is either "a machine element", or "a machine" or "a plant" for a mechanical system. Generally, a system does not function all alone but works always in connection with some other systems. As an example, a car is driven on a road, an engine is coupled to a gearbox, gears are keyed to power shafts. Thus, the Shaft, the Key, the Gear, the Gearbox, the Engine, and the Car can be defined as Engineering Systems, in general. (The concept of "system" in the last paragraph is given only for our purposes and definitions are left open to discussion for other fields of study). Accordingly, a SUBSYSTEM is any element within the system. Thus, a gear box is a subsystem within a car, or car is a subsystem within the traffic. Subsystems of the gear are, teeth, hub, involute curvature, keyway, bead, rim, hub bead and arm. The design engineers define boundaries of the system and subsystems according to their considerations on the problem. If a system is optimized, the subsystems may not be optimum designs, or if all subsystems are designed as optimum, the system constructed of these subsystems may not be optimum Even the criteria of optimization may not be the same. The system may be designed for maximum economy, but the machine elements may be designed for maximum strength with weight or volume limitations.

Another basic concept in optimization is the "*criterion*", or better "*design criterion*". The criterion solely determines the set of optimum design parameters. If the criterion is changed, the optimum parameters will change accordingly. Design criterion (pl. Criteria) is a reference plane that defines the parameters as optimum or not. Every design problem has its own criteria considering the environment and users'-oriented choices.

There may be cases where contradicting criteria are to be applied. A design product may be desired to be attractive and cheap. Hence appearance must be maximized, and cost will be minimized. All the optimization methods are developed to optimize usually one of the criteria. For the optimization of several criteria in the same design case, two approaches are suggested:

i) The existing criteria can be re-defined as a new equivalent criterion. "Weighing Functions" are defined for each criterion and a new and equivalent criterion is proposed to sum up in a new conceptual criterion function.

ii) The design criteria are usually *desirable effects*. In design cases where more than one design criterion exists, it is the designer's duty to determine the "*most significant*" design criterion. Equivalent terms are the "Most Desired" or "Most Undesired (Least Desired)" criterion. Then the single optimization criterion is replaced by the new criterion. The choice of the most significant criterion among the existing criteria is sometimes one of the most difficult steps of the optimization process. The decision is solely up to the designer, and it is within his/her responsibility. The customer is not usually qualified to make such a decision and hence ignored. Few of the customers make their choice based on safety for household appliances. The appearance, functioning and economy are claimed criteria usually. A user assumes that the design product is safe normally without knowing design details. However, human ***safety*** should be an implicit (hidden) criterion that should not be ignored at all. A similar implicit design criterion is the ***cost*** of the product. under consideration in such a case, the safety of the device is one of the design criteria.

Once the designer decides on the most significant and/or equivalent criterion, the remaining criteria in the original formulation should not be discarded but must be considered as design constraints for the problem. Design can be carried out to optimize the "most significant criterion" while remaining within the confines of the constraints, (secondary design criteria). If all the parameters of the design cannot be determined by using the "most significant criterion" then second most significant criterion should be considered, and the design should be re-optimized using the optimum parameters as invariant values after the previous optimization stage.

The design criteria are always dependent on the statement of the design problem. As an example; a spring design can be carried for minimum weight and volume for an aircraft application. The very same spring design (same spring gradient and force) can be carried out for minimum or maximum free length or external diameter on a ground application. The designer will make his/her design decisions accordingly depending on the design requirements and limitations.

Optimization is always an appealing and attractive field for designers. An inexperienced designer would like to optimize everything in his/her design project whenever he/she assigned to work on an engineering system. The designer should not rely on his solution during the early stages of the design process, and he/she should not attempt at all for an optimum design. The first goal in any design project is to satisfy the main design requirements fully. Example: Minimizing the weight of a new sewing machine which does not sew properly, is not a reasonable approach. It is certain that the selected elements of the machine will be changed soon; thus, optimization will be a useless effort. Optimization must be carried out when the whole system with all the subsystems and elements are defined properly and modelled mathematically strictly for proper functioning. Optimization should be the last stage of a design process before the final presentation and production of the physical product.

I.3 FORMULATION OF OPTIMIZATION PROBLEMS

It is possible to develop some subjective optimization methods where engineering intuition and experience is the only tool. In this section optimization problems, which can be formulated analytically will be considered.

Every optimization problem has a criterion. If all the design variables were known, it would be possible to calculate the numerical value of the criterion. The design variables (parameters) are classified as input parameters, output parameters and dummy design parameters. The input parameters are independent parameters of the design. Their values can be varied freely within certain limits. The output parameters are dependent on and determined by input parameters. The dummy parameters are used in calculating output parameters in terms of input parameters. They are used during the design optimization but do not affect the results directly. All these design parameters define a particular set.

$$\{x_1, x_2, x_3, \ldots\ldots\ldots\ldots x_n\}$$

where x_1, x_2, ...x_n are the design parameters, including input, output, and dummy parameters. Thus dimensions, material parameters, weight, natural frequency, spring gradient are design parameters.

Once the numerical value of any set $x_1, x_2, ...x_n$ is known, the design is fixed, and value of the design criterion is also known. The mathematical equation which expresses the design criterion in terms of design parameters is called CRITERION FUNCTION. Mathematically we have.

$$F = F(x_1, x_2, x_3, ...x_n) \quad (1)$$

where x_1, x_2, ..., x_n are the same parameters discussed above. The equation can be in general, in any form, linear, nonlinear, transcendental, numerical or even graphical. The form determines the method of solution.

After an optimization case is formulated, value of the criterion function F will be depended on the various combinations of the sets of design parameters. In cases where there are continuous varying parameters (like time dependent cases), behavior of the criterion functions will be varying as well. In these cases, the time parameter should be treated as an independent parameter.

The main aim in optimization is to find a unique set of design parameters which will give the best (highest or lowest) value of the criterion function for a predetermined purpose. This procedure includes mathematical optimization also; however, engineering optimization is more comprehensive and wider formulation of the simple mathematical model. All activities of this procedure to determine the best and unique parameter set is called optimization. It should be remembered that mathematical optimization is a single step in design optimization.

Decision on the feasible sets of design parameters is based on two groups of constraints which are derived from the requirements and limitations of the design process. **FUNCTIONAL CONSTRAINTS** are equations which relate the design parameters to each other. They are usually mathematical expressions for the laws of nature governing engineering laws, geometric relations, and similar equations. Functional constraints are written in terms of the design parameters like the criterion function,

$$g(x_1, x_2, x_3, ...x_n) = 0 \quad (2)$$

The number of independent functional constraints is limited by the number of design parameters. If they are equal, there is only one set of parameters which satisfy the functional constraints, therefore we cannot have an optimization problem. For a real solution, the number of independent functional constraints must be less than number of design parameters, i.e.,

$$m < n \quad (3)$$

Where m is the number of functional constraints.

Other sets of constraints are inequalities which limit the feasible or applicable ranges of the design parameters and are called **REGIONAL CONSTRAINTS**. They are necessary for physical realization of the system, for the purposes of compatibility and applicability of the design product. The number of regional constraints is not limited.

$$h(x_1, x_2, x_3, ...x_n) > 0 \quad (4)$$

The constraints can have two types of limits. If the limiting values can be changed for the benefit of criterion function or other less significant criteria, then it is called a LOOSE LIMIT. If the limiting values cannot be changed under any condition, then it is a RIGID LIMIT. Whether a limiting value is a rigid or loose limit should be determined by the designer. This decision is reached by considering the problem statement.

Considering three groups of relations (criterion function, functional constraints, and regional constraints) an analytical optimization problem can be formulated in its most general case as it is displayed on the next page.

The most difficult part of optimization problems is to put the problem statement in the mathematical form. The problem must be carefully analyzed and studied so as not to exclude any of the constraints. Even if one of the constraints is not considered, the optimum solution will not be correct or more truly, a valid one. The designer must be very careful in considering all the related constraints. Although some of the constraints are given explicitly in the problem statement, some are implicit and should be discovered by the designer. For example, strength of the machine elements is not explicitly stated in the problems, but almost every machine element must be checked for strength. Once the problem is put in the given standard form (eq.6), the rest of the work is easy. One of the optimization procedures can be applied without much difficulty. Computers can also be used after the problem is properly formulated by the designer. A list of books which discuss solution methods of optimization problems are included in the reference list.

The criterion function, functional constraints and regional constraints together define the mathematical model of the physical system. The effect of any parameter can be investigated theoretically. The success in mathematical modelling depends greatly on the completeness and validity of the three groups of relations.

Maximize

$F = F(x_1, x_2, x_3, ..., x_n)$ (5)

Minimize

Subject to:

$g_1 (x_1, x_2, x_3, ..., x_n) = 0$

$g_2 (x_1, x_2, x_3, ..., x_n) = 0$

$g_m (x_1, x_2, x_3, ..., x_n) = 0$

where $m<n$

$h_1 (x_1, x_2, x_3, ..., x_n)\ \varepsilon\ 0$

$h_2 (x_1, x_2, x_3, ..., x_n)\ \varepsilon\ 0$

$h_p (x_1, x_2, x_3, ..., x_n)\ \varepsilon\ 0$

where p has no limit. Writing the above relations in a more compact form, we obtain.

Max. or Min. $F = F (x_1, x_2, x_3, ..., x_n)$

s.t.: $g_i (x_1, x_2, x_3, ..., x_n) = 0$ $\quad i = 1,..m$ and $m<n$ (6)

$h_i (x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n) \varepsilon 0$ i = 1,…p and has no limit

I.4 PHILOSOPHY OF OPTIMIZATION PROBLEMS for DESIGN ENGINEERING

Optimization concepts in design activities should be consider4ed as a strong guidance model for the best design performance, which is optimization all alone and if applied properly may improve performance of the design product. However, a principal issue should not be oversighted. It is a known fact that all design processes must manage natural and imposed limitations and requirements. Mathematical optimization in pure theoretical modelling is straight forward process. Once a mathematical model is developed, obtaining the best (maximum and/or minimum) numerical results is not difficult in many cases. The difficulty is to reach design optima to satisfy the unclear requirements and to remain within the constraints of real-world. These issues appear as requirements in optimization and constraints (limitations) in the design problem.

Requirements are developed over well-defined numerical data.

Constraints define acceptable (and/or unacceptable) regions of the design process. However, it should be emphasized that requirements and constraints are not on the design processes, but they are to be implemented on the design product. Since the design products are normally unknown/undefined existence of goods, property, and simply existence. Existence of design items are imagination at early design process, physical realization is achieved only after a proper design[2] activity.

Human requests and willingness of best and/or better products, their implemented and associated engineering concepts of existence, human desire for better and/or best accessible comfort are the basic concepts in the roots of optimization activities. As this is true for consumer society, similar preference requests are observed on the industrial side where the producers claim to reach economically producible products to sell the public society.

The above concepts are converted to and represented by mathematical expressions in the book, The link and process of conversion from abstract concepts and imaginary products to real physical items to change human comfort is the design process, attempts to make the design and design process the best or better than others, with a mathematical modelling, are optimization.

1.5 CHAPTER CLOSURE

It should be clearly understood that the optimization procedures are not a necessary step in engineering design. However, if the designer plans to develop a unique design product with better/best features, he/she must apply optimization techniques strictly. It's the only way for optimum design. The bad news is that design is a conceptual activity whereas optimization is a mathematical procedure. Here it is within the responsibilities of the designers to convert the physical product (design) model into a mathematical

[2] Remember to add 'engineering' to 'Design activity' or 'design process' for real physical items after a mind driven imagination activity. A clear separation is known to exist between engineering design and design engineering. In practice.

expression. Good news here are that design product is in the designer imagination at this stage, hence a shift into mathematical model may be smooth and reliable for the possible physician design product.This the main goal of this book is to introduce and practice designer with simple design problems to practice theoretical procedures. Later we hope that today's students may have chances to apply optimization approaches for professional design cases. Remember always that the basic optimization process and procedure is exactly the same, except that the number of design parameters is significantly higher in the professional world than the students' level work.

My first suggestion in this respect is to double or even multiply the design problem. Explicitly, you may identify smaller units of the larger design product and then define smaller optimization problems. Separation procedure requires expertise in a narrow field of science and/or engineering. By this way, you may define new but simpler problems Then you may apply two or more optimization approaches and combine them later.

PART II
MATHEMATICAL METHODS of OPTIMIZATION PROBLEMS for DESIGN ENGINEERING STUDENTS

Chapter II 1
Chapter II 2
Chapter II 3
Chapter II 4
Chapter II 5
Chapter II 6

CHAPTER II
CONCEPTS and PHILOSOPHY of OPTIMIZATION

II.1 INTRODUCTION

Optimization can be visualized in n+1 dimensional space (n design parameters and criterion function F form the (n+1) space). The constraints of the design problem bound a region in space and any point within this region is a feasible point of the problem. The optimum point is found by a search among these points. Mathematical form of criterion function and constraints determine the searching method. If we have linear relations, then the corner points of the intersecting lines and planes will give us feasible points. If non-linear relations are under consideration, curved surfaces should be investigated. By mathematical techniques, the designer moves on these curvatures and finds the maximum or minimum point.

The selection of the optimization method is determined by the nature of the problem. *Figure II.1* gives an optimum design probe schema. It gives only the basic rules, and further methods should be decided by the designer.

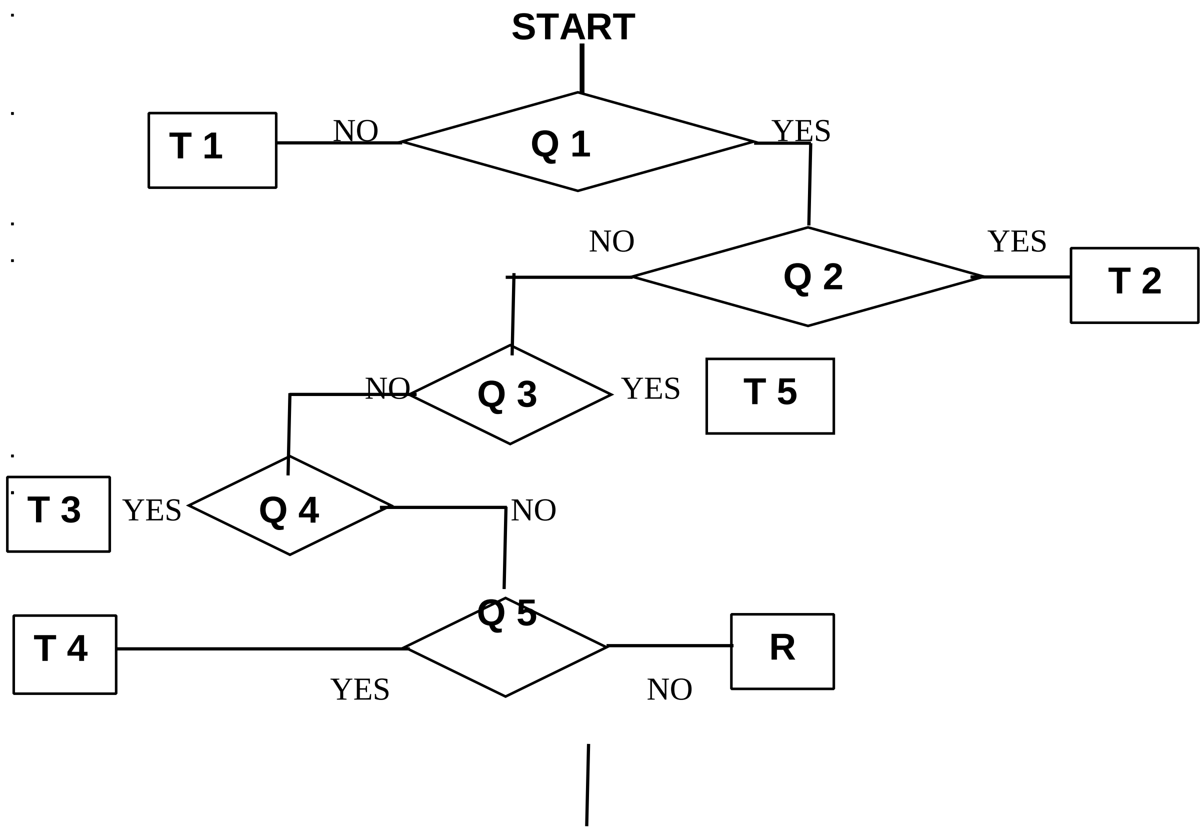


Figure II.1 Optimum Design Probe Schema

Q.1 Is it possible to develop a mathematical model for the physical system?
T.1 Apply Logical Optimization Techniques
Q.2 Are the constraints and criteria function linear?
T.2 Apply Linear Programming Techniques
Q.3 Is there any regional constraints?
T.3 Apply method of differentiation
Q.4 Is the criterion function differentiable?
Q.5 Is the method of calculus of variations applicable?
T.4 Apply calculus of variations techniques
R Reformulate the problem.
T.5 Apply engineering optimization approaches.

It is possible to classify the optimization methods in several ways. Constrained and Unconstrained Optimization method of general classification. Numerical and analytical Optimization is another way. For a comprehensive general classification, mathematical backgrounds may be important and determinative. However, our main concern in the book is engineering application and therefore engineering methods. Hence, we prefer to classify similar types of type of applications which display similar performance behaviors.

The following pages group the similar procedures of optimization for a practical purpose.

II.2 METHOD OF SIMPLE DIFFERENTIATION

Some of the optimization problems without any regional constraints can be solved by simple differentiation. Consider a single variable function F(x) as shown in Figure II.2. This is a criterion function with one design parameter. It is known that points B, C and E can be computed by simply differentiating F(x) and setting it equal to zero. This is because the derivative $\left(\frac{dF}{dX}\right)$ is the slope of F(x) and the slope is horizontal at the points B, C and E. The roots of the equation.

$$\frac{dF}{dX} = 0$$

gives x_B, x_C and x_E where x_A is a relative minimum x_B is a relative maximum and x_E is a saddle point. The word "relative" is used to indicate that it is only a local optimum and the end points of the curve could have larger or smaller values than the maximum point (maxima) or minimum point (minima) respectively. Several cases are illustrated in *Figure. II.2.* The second derivative (d^2F/dx^2) indicates whether the computed root is a maxima or minima. If it is a negative value, the root is a maximum, If it is a positive value we have a minimum value. The saddle point gives zero for the second derivative.

This procedure can be extended to n variable functions.

$F = F(x_1, x_2, ...,x_n)$

The maxima or minima points (together they are called extremum) can be determined by taking partial derivatives of the given equation with respect to each of the variables. These equations are equated to zero, thus ‘n’ independent equations are obtained.

$\frac{aF}{ax_1} = 0$ and $\frac{aF}{ax_2} = 0$ n variables $\{x_1, x_2, ..., x_n\}$, n equations

$\frac{aF}{ax_n} = 0$

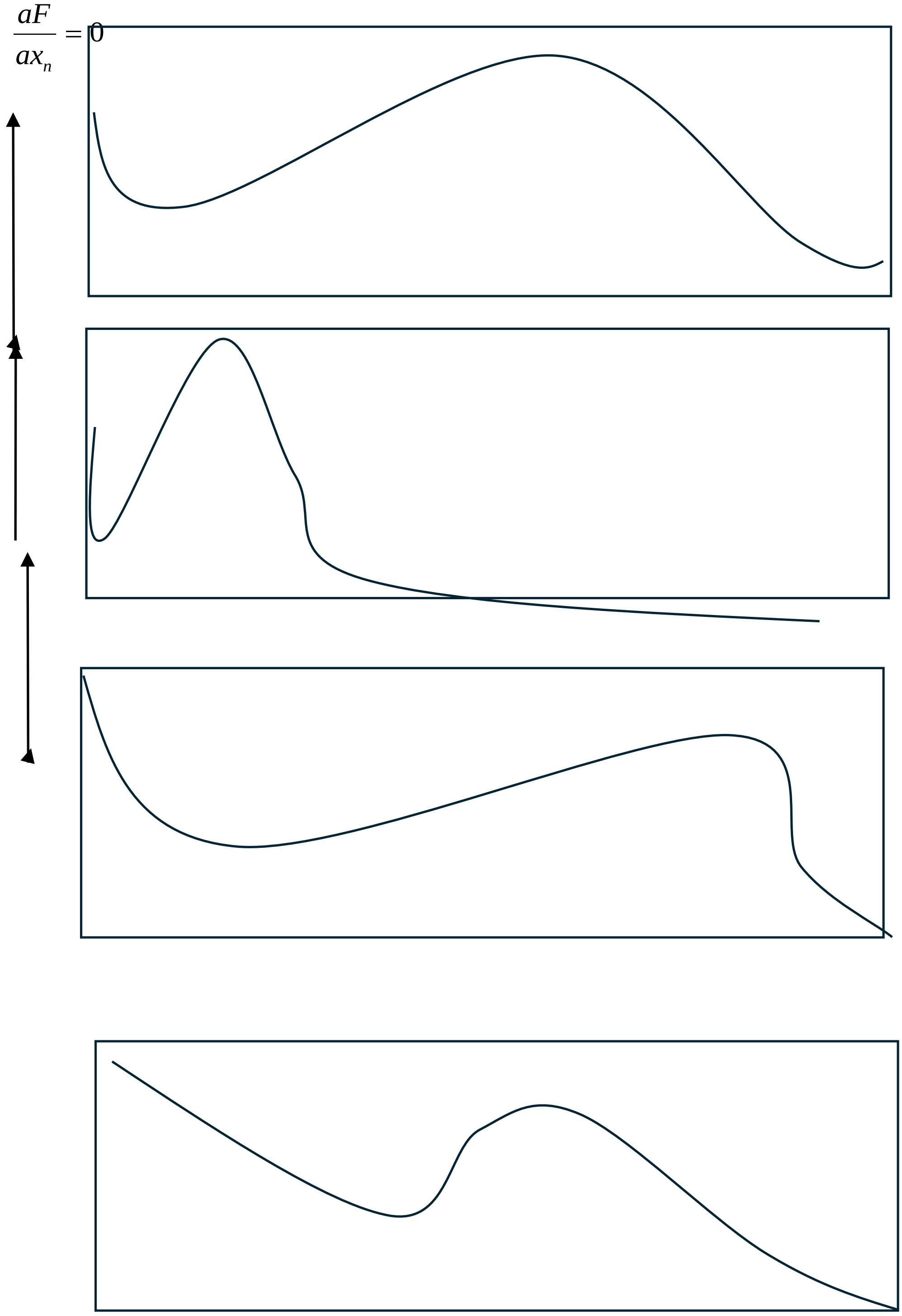


Figure II.2 Arbitrary forms of “one-variable function” F(x)

Simultaneous solutions of these equations give the extremum point(s). However, it should be remembered that these point(s) may not be absolute maxima or minima. As it is shown in one variable must be checked to determine whether it is a relative or absolute extremum.

Further, suppose that, besides the n variable function, we have several functional constraints.

Max. Or Min. F=F (x_1, x_2, x_3, ...x_n)

s.t. g_i (x_1, x_2, x_3, ...x_n) = 0 i = 1, m and m<n

In this case, the constraint equations must be included in the solution to have feasible point. In other words, the design variables x_1, x_2, ...x_n which satisfy the functional constraints must be considered in the derivatives.

The functional constraints can be eliminated by solving a common parameter between the criterion function F and a functional constraint. The equivalent of each of these variables solved from any g function is substituted into the F function. Mathematically.

Take any g (x_1, x_2, ...x_{r-1}, x_r, x_{r+1}, ..., x_n) = 0

Solve for x_r, where x_r is any variable which explicitly appears in

F = F(x_1,x_2,..........., x_r, ...,x_n),

x_r = r(x_1,x_2, ...x_{r-1},x_{r+1}, ..., x_n)

Substitute this equality into F, we have a new function; F’ ,

F’ = F’(x_1,x_2, ...x_{r-1}, x_{r+1}, ...x_n

Hence a functional constraint and a design variable, are eliminated from the problem formulation. Since there are m functional constraints in general, then m design variables can be eliminated. The final form of the criterion equation is a function of (n-m) variables.

F’ = F’ (X_1, X_2, ..., X_{n-m})

where X_1, X_2, ...X_{n-m} are the parameters which are not eliminated. Then we have a new function F’ which is not constrained (does not have any constraint) and the extremum points can be found by partial derivatives as it is discussed above.

II.3 METHOD OF LAGRANGE MULTIPLIERS

The applicability and limitations of this method are the same as the method of simple differentiation. The regional constraints are not considered in mathematical solutions. We start again in the mathematically proper form of the problem. Mathematically Total Derivative of the criterion function is considered.

F = F(x_1,x_2,x_3, ...,x_n)

$$dF = \frac{aF}{ax_1}dx_1 + \frac{aF}{ax_3}dx_3 + ... + \frac{aF}{ax_n}dx_n$$

To have an extremum

dF = 0

must be satisfied. If there were no functional constraints, i.e., if p=0, then

x_1, x_2, ... x_n would be independent. Thus, dx_1, dx_2, ...dx_n would be independent and therefore their coefficients would be equal to zero simultaneously.

$$\frac{aF}{ax_1} = 0, \quad \frac{aF}{ax_2} = 0, \quad ..... \quad \frac{aF}{ax_n} = 0$$

This is discussed before as an unconstrained problem. Since there are m functional constraints which relate the design parameters, the coefficients in the total derivative are not zero anymore. The functional constraints must be involved somehow in the solution. Lagrange has devised a method where constraints are included in the criterion function. He considered the total derivatives of the criterion function and functional constraints together and summed all of the equations.

$$dF = \frac{aF}{ax_1}dx_1 + \frac{aF}{ax_2}dx_2 + ... \frac{aF}{ax_n}dx_n = 0$$

$$dg_i = \frac{ag_i}{ax_1}dx_1 + \frac{ag_i}{ax_2}dx_2 + ... + \frac{ag_i}{ax_n}dx_n = 0$$

i = 1, m and m<n

Since the units for *d*F and each dg_i are different, for a valid summation we must introduce some dummy coefficients for each dg_i. These are called Lagrange Multipliers and usually abbreviated as λ_i. Then we have.

$$dF + \sum_{i=1}^{m}\lambda_i dg_i = \left(\frac{aF}{ax_1} + \sum_{i=1}^{m}\lambda_i \frac{ag_i}{ax_1}\right)dx_1 + \left(\frac{aF}{ax_2} + \sum_{i=1}^{m}\lambda_i \frac{ag_i}{ax_2}\right)dx_2 + ... + \left(\frac{aF}{ax_n} + \sum_{i=1}^{m}\lambda_i \frac{ag_i}{ax_n}\right)dx_n = 0$$

Since all of the functional constraints are included in this equation, the differential parameters dx_1,dx_2, ..., dx_n are all independent now and, therefore their coefficients must be equal to zero.

$$\frac{aF}{ax_j} + \sum_{i=1}^{m}\lambda_i \frac{ag_i}{ax_j}\frac{ag_i}{ax_j} = 0$$ for j=1, n

There are n such equations, but there are n design parameters plus m artificial parameters λ_i (Lagrange multipliers). Considering constraints also, we have (n+m) equations. Thus, the number of unknows and available equations are the same hence a simultaneous solution will give us the optimum point.

If the above equation is examined carefully following arrangement is evident since all λ_i are independent of x_j.

$$\frac{a}{ax_j}\left[F + \sum_{i=1}^{m}\lambda_i g_i\right] = 0$$

Also note that the functional constraints can be written in the following form:

$$g_i = \frac{a}{a\lambda i}\left[F + \sum_{i=1}^{m}\lambda_i g_i\right] = 0$$

The terms within the above brackets are the same and it is called LAGRANGIAN.

$$L = F + \sum_{i=1}^{m}\lambda_i g_i$$

It is apparent that the new criterion function is L and functional constraints g_i multiplied by a Lagrange multiplier. The Lagrange (L) is a function of n+m variables.

$$L = L(x_1, x_2, x_3, ...x_n, \lambda_1, \lambda_2, \lambda_3, ...\lambda_m)$$

and L has no functional constraint. Hence total derivative vanishes at the optimum point and coefficients of variables should be taken equal to zero.

$$\frac{aL}{ax_1}=0,\frac{aL}{ax_2}=0,........\frac{aL}{ax_n}=0,\frac{aL}{a\lambda_1}=0,\frac{aL}{a\lambda_2}=0,.....\frac{aL}{a\lambda_m}=0$$

where the last m equations are original functional constraints. Solving (n+m) equations for (n+m) parameters, we can obtain the extremum, point.

One should note that the functional constraints g_i added into the criterion equation are all equal to zero, therefore optimum values of L and F are the same numerically.

Apparent advantage of the method of Lagrange multipliers is that it is straightforward and numerical methods aided by computers can be applied easily.

The methods of simple differentiation and Lagrange Multipliers can be applied directly to the problems without any regional constraints. When regional constraints exist, then the solution should be conducted as if there is no regional constraint. If the results satisfy the regional constraints, then it is an optimum and feasible solution. If not, the end points are the possible feasible and optimum solutions. Depending on the effect of parameters on the criterion function end points are decided as extremum points. This will be illustrated in problem. The same problem can be solved by the method of Lagrange Multipliers and the same numerical answers should be obtained. This is left to the reader.

II.4 GRAPHICAL METHODS

Purely graphical methods are seldom finding application in practice since the number of parameters which can be used is limited to 1 or 2 at most. When it is a three-parameter problem it is impossible to apply graphical methods on the 2-dimensional plane of paper.

Optimization by graphical means is based on plotting the criterion function and the functional and regional constraints. The criterion function with variable design parameters forms either parallel or crossing or at least similar curves. These are illustrated in *Fig. II.3* for one dimensional case. The value of F increases in a preferred direction depending on the form of the equation. The number of these criterion functions is limited by the functional and regional constraints when they are plotted on the same figure. The regional constraints exclude a certain area in F vs x plane. The excluded area is not feasible for optimum solution. The regional constraints define a curve therefore the optimum point must be necessarily on this curve. This is illustrated in *Fig. II.4*. The feasible region and optimum point become apparent when all the curves are plotted on the same figure.

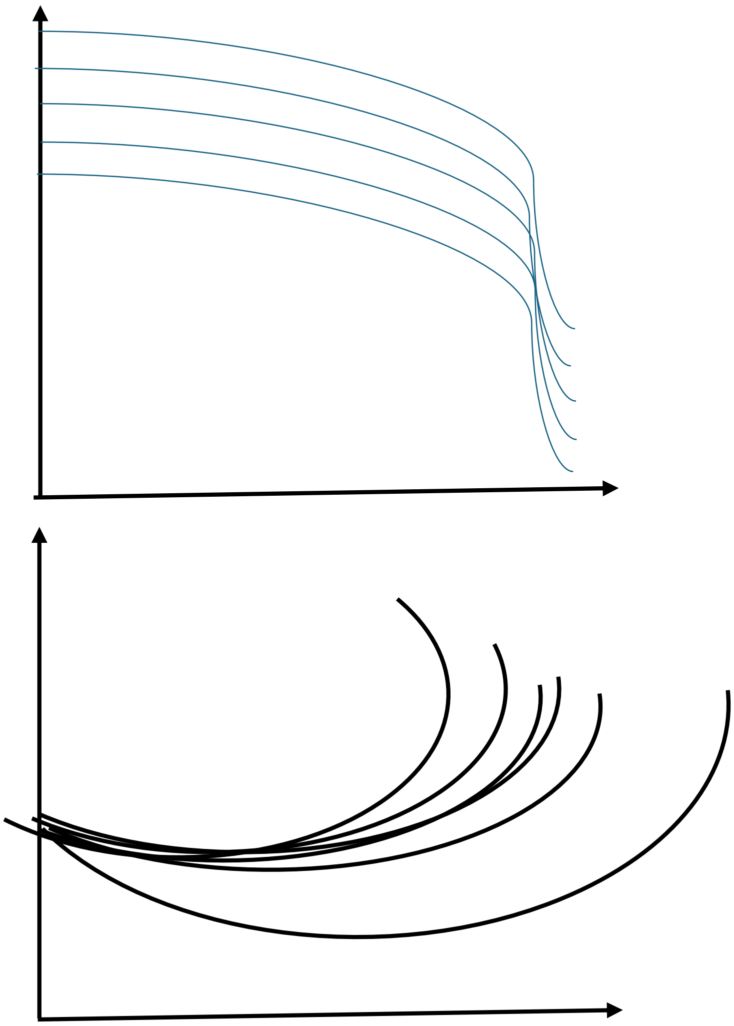


Figure II.3 Parametric forms of mathematical equations.

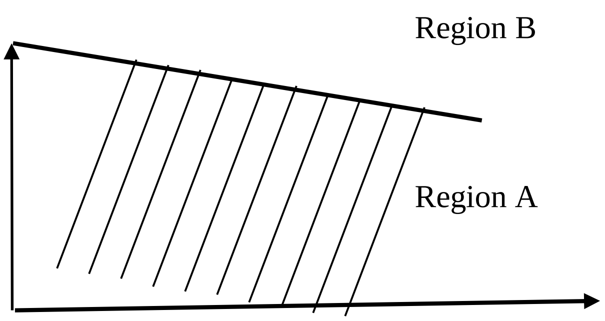


Figure II.4 Graphical representation of Feasible Regions.

II.5 LINEAR PROGRAMMING

It was shown that any optimization problem can be formulated in three groups of equations.

Max./Min. $F=F(x_1,x_2,......, x_n)$

s.t. $g_i(x_1,x_2,.........., x_n) = 0$ $i = 1, m$ $m<n$

$h_i(x_1, x_2,........., x_n) \varepsilon 0$ $i = 1, p$ p has no limit.

There is a vast group of problems where the above relations are all linear. Thus, the general form of the optimization problem is in the following form:

Max.Min. $F=f_1x_1+f_2x_2+f_3x_3+..........+f_nx_n$

s.t. $g_{i1}x_1+g_{i2}x_2+...........+g_{in}x_n+c_i = 0$

$i = 1, m$ $m<n$

$h_{i1}x_1+h_{i2}x_2+............+ h_{in}x_n+k_i\varepsilon 0$

$i = 1, p$ p has no limit.

Where C_i and K_i are plain numerical constraints.

Further the problem can be so arranged that all the design parameters can have positive values only.

$x_1 \varepsilon 0,$ $x_2 \varepsilon 0, ...x_n \varepsilon 0$

This is called a nonnegativity condition.

The above problem can be re-formulated in the matrix form. The criterion function is a product of two matrices. That is

$F = [f]^T [x]$

Where $[f]^T$ is the transpose of [f]

$$[f] = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \dots \\ f_n \end{bmatrix} \quad \text{and} \quad [x]=\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \dots \\ x_n \end{bmatrix}$$

Similarly functional constraints can be written in matrix form.

$[G][x]=[C]$

Where.

$$[G]= \begin{bmatrix} g_{11} & g_{12} & \dots & g_{1n} \\ g_{21} & g_{22} & \dots & g_{2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ g_{m1} & g_{m2} & \dots & g_{mn} \end{bmatrix}$$

and

$$[C]= \begin{bmatrix} C_1 \\ C_2 \\ .. \\ \\ Cm \end{bmatrix}$$

The regional constraints transform.

$[H][x] \geq [K]$

$$[H]= \begin{bmatrix} h_{11}\ h_{12} \ldots h_{1n} \\ h_{21}\ h_{22} \ldots h_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ H_{p1}\ h_{p2} \ldots h_{pn} \end{bmatrix} \quad \text{and} \quad [K]= \begin{bmatrix} -K_1 \\ -K_2 \\ -K_3 \\ \\ -K_p \end{bmatrix} \ldots$$

and non-negativity conditions in matrix form are:

$[x] \varepsilon [0]$

Considering the above matrix equations, the optimization problem can be written in terms of four matrix relations.

Max. Or Min. $F = [f]^T [x]$

s.t. $[G][x] = [C]$

$[H][x] \varepsilon [K]$

$[x] \varepsilon [0]$

Any optimization problem which is formulated in this form can be solved by a method which is called Linear Programming. It is possible to show the method of solution by matrix algebra, but it is beyond the scope of this text. Two dimensional problems can be solved by purely graphical means, and it is very useful. If the number of parameters is more than 2, there are some methods (most applicable one is Simplex Tableaux) to obtain the optimum point. Although hand calculations are possible it is time consuming and prone to make calculation errors. Instead, computer techniques are preferred. Almost all companies provide ready-to-use Linear Programming packages supplied with their computers. The only work to be done by the user is to supply the numerical coefficients; in other words, the matrices [f], [G] and [H] must be given as the input data. The optimum point (if exists) and some information is given as output by the computer.

Some optimization problems cannot be readily put in a format suitable for Linear Programming. The most common headache for mechanical design applications is the non-negativity conditions. Generally, we have:

$M\ \delta\ x_r\ \delta + M$

where x_r is any design parameter and M is a positive number, including □. It is possible to define two new parameters x_r' and x_r'' such that:

$$x_r = x_r^1 - x_r^{11}$$

with the condition.

$$x_r^1 \geq 0, \qquad x_r^{11} \geq 0$$

Therefore, x_r is replaced by $x_r^1 - x_r^{11}$, the linearity of the relations is still maintained, and non-negativity conditions are satisfied.

If the criterion function or constraints are not in linear form, we can either approximate them as linear equations in certain ranges (see Fig. II.5) or apply some mathematical tricks. To illustrate this suppose we have a function as:

$$F = x_1^a 1 x_2^a 2 x_3^a 3 ....x_n^a n$$

where a_1, a_2, ...a_n are constants. This is an exponential equation and cannot be used in Linear Programming. If we take logarithm of both sides, we obtain a linear equation in terms of logarithms of parameters.

$$\ell nF = a_1 \ell n x_1 + a_2 \ell n x_2 + ... + a_n \ell n x_n$$

Defining new parameters as

$$F^1 = \ell nF$$

$$x_1^1 = \ell n x_1, \quad x_2^1 = \ell n x_2, ...x_n^1 = \ell n x_n$$

we have

$$F^1 = a_1 x_1^1 + a_2^1 x_2^1 + ...a_n x_n^1$$

This is a linear equation, and the method can be applied without any difficulty. Some other mathematical tricks, depending on the user's ingenuity, can be found and applied successfully.

It was stated that two-dimensional Linear Programming problems can be solved by graphical methods. This will be illustrated now. Consider a two-parameter problem in its most general form.

Max. Min. $F = f_1 x_1 + f_2 x_2$

$$g_1 x_1 + g_2 x_2 + C = 0$$
$$h_{11} x_1 + h_{12} x_2 + K_1 \geq 0$$
$$h_{21} x_1 + h_{22} x_2 + K_2 \geq 0$$
$$...................................$$
$$h_{p1} x_1 + h_{p2} x_2 + K_p \geq 0$$

s.t.: $x_1 \geq 0, x_2 \geq 0$

All these relations can be shown on a 2-dimensional plot. Firstly, non-negativity conditions exclude all the quadrants except the first. Consider criterion function F in the first quadrant. Since f_1 and f_2 are known numerically, for any arbitrary value of F, a straight line can be drawn in x_1-x_2 plane (Fig, II.6). F_1, F_2, ...are such lines and they are naturally parallel. The value of F increases in the direction of arrow depending on the signs of f_1 and f_2. That is:

Either $F_1 > F_2 > F_3 > F4$ ..... or $F_1 < F_2 < F_3 < F_4$ is true.

Hence, the optimum (Maximum or Minimum) point can be reached by moving in the direction (or against it) of arrow. How far we will move is determined by the constraints.

The functional constraint defines a line in x_1-x_2 plane and if it is satisfied by the design parameters, it defines a feasible region along the line segment AB only. Any point on this line is a feasible point and the optimum point is one of these (Fig.II.7).

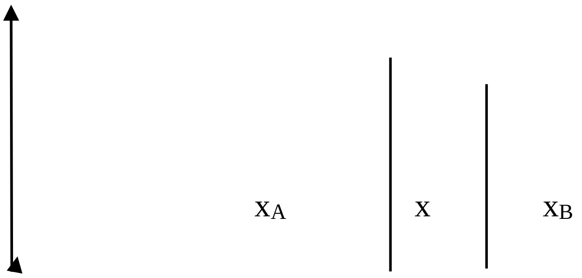


Figure II.5 Approximating a function in linear form in region $x_A < x > x_B$

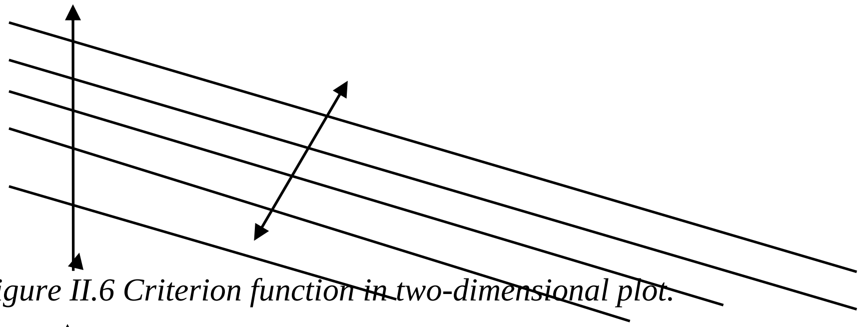


Figure II.6 Criterion function in two-dimensional plot.

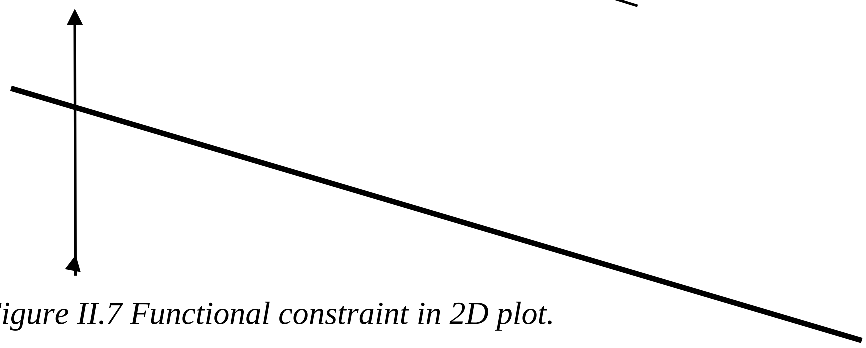


Figure II.7 Functional constraint in 2D plot.

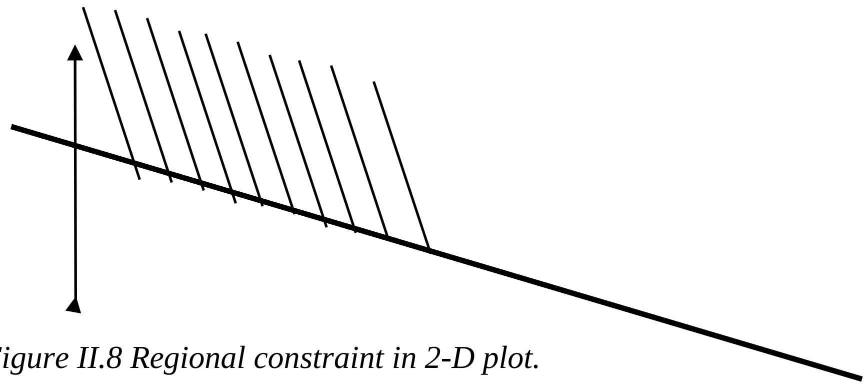


Figure II.8 Regional constraint in 2-D plot.

Consider the regional constraints in x_1-x_2 plane. Let us take the first,

$$h_{11}x_1 + h_{12}x_2 + K_1 \leq 0$$

Depending on numerical values and signs of h_{11}, h_{12} and K, the inequality relation divides the first quadrant into two regions (Fig. II.8). The points in the hatched region do not satisfy the inequality therefore they are not feasible points. Any point in the feasible region may be an optimum point. When the regional constraints are plotted on the same figure, we obtain the common feasible region defined by all the regional constraints (Fig. II.9). Further imposing the functional constraint on this figure (Fig. II.10), we have the final feasible region as a line segment in x_1-x_2 plane. Usually, such problems do not have functional constraints, thus similar regions as shown in Fig. II.9 are obtained.

To find the optimum point, it is sufficient to draw the parallel lines representing the criterion function. The maximum or minimum point will be on one of the corners of the feasible region.

Applications of Linear Programming problems can be summarized in 4 groups.

1- Alloying: Several elements with different costs and properties are considered and Linear Programming is applied to find minimum cost with the given constraints. Metallurgical alloying, liquid blending, ore combinations and cattle feed mixing are examples.
2- Job Allocation: The machines (capacity and time) labor, and raw materials are arranged such that the total profit from production of several items is maximized.
3- Production Scheduling: The production rates of several products are determined by considering Supply, Demand, Inventory and Storage facilities.
4- Transportation: The best routes and schedules are determined for all sorts of vehicles.

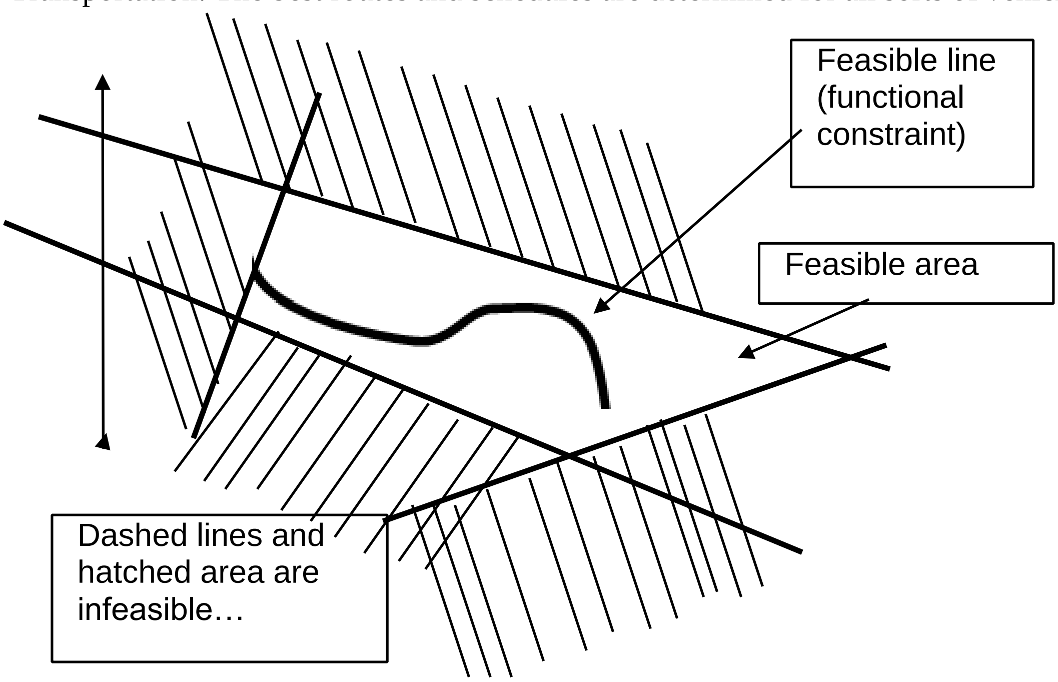


Figure II.9 Several Regional Constraints and imposed functional constraint.

II.6 AN APROACH TO SOLUTIONS OF OPTIMIZATION PROBLEMS FOR MECHANICAL ENGINEERING APPLICATIONS

An optimization method, applied particularly in mechanical engineering design, is described by Johnson in his several books and papers (14, 15). It is a clearly described procedure for optimization problems rather than being a mathematical optimization method. The steps of optimization are clearly defined and confusion in multi variable problems are reduced by a systematic approach. Several interesting examples are solved by the described procedure. We think that the procedure given by Johnson can be applied to any optimization problem and the mathematical methods described in the previous sections can be used within the procedure. In the next sections, the procedure will be described with some small changes. But before this, there are several concepts which must be explained clearly.

Any machine or machine element has three basic properties.

1. Functional Requirement Properties.
2. Desirable or Undesirable Effects.
3. Uniqueness.

Now, we shall study these properties:

1. **Functional Requirement Properties**: Any machine or machine element is designed for a certain purpose. It must perform its functions in this predetermined way. Consider a spring, which is designed to exert a force F. Then the force F is a functional requirement. It is a property that we expect from the spring. Similarly, a shaft must transmit torque, a gearbox must transmit the power at a certain input and output speed, a cam must give a predetermined displacement to its follower, and finally a structural number (leg of a table in its simplest form) must carry a certain weight. The functional requirements must be satisfied in a predetermined way, as predicted by the designer.

2. **Desirable and Undesirable Effects**: In designing the machines and their elements to satisfy the functional requirements, there are some properties which are unavoidable for a physical system. For example, a spring cannot be designed with an imaginary material and since all of the real materials have a certain upper limit for the stresses, then the stress is an unavoidable property. Similarly, any real material has a weight under the gravitational field, and it is again unavoidable. Deflection, space occupancy, vibration and natural frequency are other properties for any simple helical spring. Some of these properties are desirable and some are undesirable depending on the application. For example, weight in some applications can be desired whereas in some others it is not desired. Other examples for desirable and undesirable effects are surface quality, machinability, length, diameter, appearance, comfortability stability, volume, cost, production time etc. All these effects, if significant, must be considered in any optimization problem.

3. **Uniqueness**: Any machine or machine element can be defined completely if all the dimensional parameters and the materials parameters are specified. This statement can be put in reverse order; any machine or machine element can be specified completely once all the material parameters and geometrical parameters are known. The geometrical parameters are length, diameter, width, height, tolerances etc., and the material

parameters are Elasticity modulus, Yield Point, Weight density, Unit Cost, Poisson's ratio etc.

In any design problem statement, the above properties are either specified numerically or their allowable limits are given. Thus, a diameter can be required to lie between, say, 5 mm and 15 mm or a spring force can be stated to have values 50 kg ± 10 kg. Λ natural frequency can have any value between 0 and □ in its widest range. Usually, all the properties and their corresponding parameters have lower and upper limits. If these lower and upper limits are required, it is called a RIGID LIMIT, if slight changes on these limiting values are permissible, then these are called LOOSE LIMITS. Loose limits are useful when it is possible to obtain better solutions for the design problems with slight changes in the limiting values. Apparently, these "Slight changes" should not make the solution impossible or impractical.

During the mathematical formulation of the problem, the above stated properties are expressed in terms of four groups of parameters.

1. Functional Requirement parameters,
2. Desirable and Undesirable effect parameters,
3. Geometrical parameters,
4. Material parameters.

Functional Requirement parameters include the mathematical equivalents of the functional requirement properties. They are usually external to the machine or machine element. Mostly they express the relation between the neighborhood machines. (Torque of gear is transmitted from a shaft, and shaft receives the power from an electric motor). Thus, before starting a design, the functional requirement parameters must be known and determined due to the external effects. Sometimes the resulting values of these parameters are defined as the effect of the design to the design to the surroundings. In both cases, it is considered as the overall reaction to the next machine or element (system).

Desirable and Undesirable Effect parameters are determined terms of design parameters during the design process. They do not affect the design of the other machines or elements directly. Only a cumulative effect on the most general system is possible. For example, the total cost of a machine is the sum of the cost of each machine element.

Geometrical parameters are usually independent of each other, but they determine the previous two groups of parameters. If they are limited, the geometrical parameters are dependent on each other. There may be also limiting values on the geometrical parameters. Usually, the designer finds great freedom in changing the geometrical parameters independently. But he must be careful not to violate the limits. The geometrical parameters are mostly continuous variables like diameter or length of a shaft to be turned on a lathe. But sometimes they have stepwise variation. The standardized machine elements (Roller bearings, bolts, screws, gears (partially) and others). Have some preferred dimensions and the designer must make his choice among these values.

Material parameters are described in discussing material properties. Their common property is the interdependence of all the material parameters. If some of the material parameters are determined (say, yield point) the number of alternative materials is limited. In any design application, the number of available materials is limited and not more than 10 or a close figure. If the material name is known, then all of the material parameters are known. Specifying C-1040 with its commercial name is equivalent to specifying all its related parameters.

Most of the parameters in these four groups must remain within some practical limiting is called a feasible parameter set. One of these sets is the optimum design. The optimization is based on one of the functional requirements and/or desirable and undesirable effects.

There are three groups of mathematical relations in any optimization problem as it was discussed before. The names given previously were adapted from mathematical theories. We shall now introduce and use names which are more proper for engineering design.

The Primary Design Equation (PDE) corresponds to the criterion function. It is the most significant functional requirement, Desirable effect, or Undesirable effect. The choice of maximization and minimization depends on the problem. The most common applications are given below.

Minimize: Cost, Weight, Volume, Deflection, Natural Frequency, Length, Speed, Instability, Force, Area, Stress, ...

Maximize: Power Transmission Capability, Energy Storage, Speed, Natural Frequency, Weight, Length, Stability, Force, Area, Safety Factor, ...

The designer must be careful that some negative properties like cost must never be maximized and positive properties like efficiency, power transmission capability must never be minimized.

Besides the nature of the property to be optimized, the application also determines the choice. Consider a shaft, exactly similar in two different applications like Stone Crashing machine and Aircraft. The criterion of shaft design for the Aircraft will be “minimum weight”. The weight is not important at all in the first case and perhaps torque transmission capability will be the design criterion.

Subsidiary Design Equations (SDE) are all the equality relations other than the Primary Design Equation. These were called previously as functional constraints.

Limit Equations are the regional constraints of mathematical methods.

All the equations should be arranged such that the parameters stated above will appear as independent parameter groups. This is not always possible, and some mathematical tricks (like defining new parameters) is applied in such cases. These are illustrated in the examples to be given later.

II.7 PROCEDURE OF OPTIMIZATION (9 STEPS)

Any optimization problem derived from engineering applications and its solution can be modelled in 9 steps[3].

Step 1: Draw a free hand sketch of the system and show all of the related parameters. Select the independent parameters which will uniquely define the geometry. If a choice exists, select parameters which are either specified as functional requirements or limited to permissible values.

[3] 9 steps may be valid only for engineering based design problems.

Step 2: Decide on the most significant criterion for optimization and write the mathematical expression. This is the Primary design equation (PDE). If possible, write this equation in terms of parameter groups.

Step 3: Write the equations for all other significant functional requirements and desirable and undesirable effects. These equations are Subsidiary Design Equations (SDE).

Step 4: Write the mathematical limit equations for all the parameters. Indicate rigid and loose limits.

Step 5: Eliminate a common design parameter from the Primary Design Equation for each Subsidiary Design Equation. Re-write the remaining Subsidiary Design Equations and Limit equations in terms of the remaining design parameters. If choice exists eliminate unlimited and unspecified parameters. Do not eliminate material parameters since they are dependent on each other and have limited ranges. Dimensional parameters limited within finite ranges should not be eliminated, if it is not necessary. At the end of this step, the original design equation is written in terms of limited parameters and geometrical parameters. This equation is called Develop Primary Equation (DPDE).

Step 6: From the Develop PDE, obtain variation of the design criterion w.r.t. each of the parameters (except material parameters). Draw rough sketches indicating the general trend of the criterion within the feasible range of the parameters.

Step 7: Considering the variations determined in the previous step, obtain optimum design quantities. If it is necessary, apply the mathematical optimization techniques. The set of optimum parameters defined at this step must define a unique system with the design criterion as optimized.

Step 8: The only remaining parameters are material parameters, and they can be independently isolated from the other parameters before the 7th step. Select the optimum material by considering the Material Selection Factor and available materials for the design problem. Remember that the optimum material must be one of the available materials and material parameters are dependent on each other.

Step 9: Determine optimum values for the eliminated parameters by using the known optimum parameters.

II.8 OPTIMUM SHAPE DESIGN

Sometimes it is required to determine the optimum shape of machine element with a particular criterion. Examples of this sort are the shape which gives minimum circumferential length for a fixed area, or the shape of a connecting arm for maximum strength. Solution of such problems is not very simple and usually requires the application of advanced mathematical techniques. Sometimes numerical methods are applied.

Optimization problems for optimum shape design are generally solved by two methods.

i) Direct methods
ii) Method of Calculus of Variations

In this section, the direct methods will be studied. The value of criterion is not usually unlimited, and the optimum point is obtained at either upper or lower limit. For example, cost is always required to be minimized. Thus, the ideal optimum solution is zero, which is never possible. There must be a lowest limit on the cost which is defined by some factors necessary for the physical realization of the system. Strength is such a principal factor. In other words, to have the necessary minimum strength for a, say, beam, a certain amount of material must be used and that is equivalent to the cost. If we can design the system such that the effective factors are kept at their limiting values for all of the possible conditions, then we obtain the “best solution” and it is an optimum design. Mathematically some of the regional constraints are converted to functional constraints by considering equality sign instead of inequalities. This cannot be done for all regional constraints since in that case, no feasible solution can be obtained. Consider an optimization problem, mathematically stated in its proper form as it is discussed before.

$$Opt.F = F(x_1, x_2, x_3, ... x_n)$$

$$s.t. g_i(x_1, x_2, x_3, ..., x_n) = 0$$ i = 1,..m, m<n

$$h_i\ (x_1, x_2, x_3, ..., x_n) = 0 h_i\ (x_1, x_2, x_3, ..., x_n) = 0$$

i = 1, p, p has no limit

In usual applications of this sort, m=0 and p=1 for a practical solution. The boundary of the system defined by h^1 function is the only feasible solution and it is optimum.

II.9 OPTIMUM DESIGN BY THE METHOD OF CALCULUS OF VARIATIONS

There is a group of optimization problems which have the criterion function in integral equation form.

$$I = \int_{x_1}^{x_2} F(x, y, y^1) dx$$

where I is the criterion function, x is the design parameter, y=y(x) is a mathematical function and y^1=dy/dx. The integrand function F is known explicitly, and it is required to determine y(x) which optimizes the integral value I. By mathematical methods it is possible to show that the optimum point is obtained when the Euler-Lagrange condition is satisfied.

$$\frac{aF}{ay} - \frac{d}{dx}\frac{aF}{a_y 1} = 0$$

Knowing F explicitly, one obtains a differential equation which can be solved to determine the optimum function y(x).

II.10 OPTIMIZATION BY NUMERICAL METHODS

When the number of parameters is large, it is usually difficult to apply manual computation. Numerical methods in such cases help us to find the optimum solutions. It is a broad area of study and beyond the scope of this text. Several methods, like. Exhaustive search, area elimination, Fibonacci search, Golden section method, Grid search and Gradient search are discussed in several other books and some of the important references are given in the reference list.

II.11 METHOD of DYNAMIC PROGRAMMING

Dynamic programming is developed to solve a special class of problems where multi-stage decisions are required. For example, consider a problem where we start from the first stage and reach to Nth stage (Fig. II.11). There may be several alternative points to start on at the first stage and there may be several alternative end points at the final stage. Further, the intermediate stages may have many different alternatives. Since in each stage, we must select a single alternative, then the successive selection of these alternatives will change the value of the criterion function. The types of problems solved by dynamic programming are Network problems, resource Allocation and Reliability problems or similar cases.

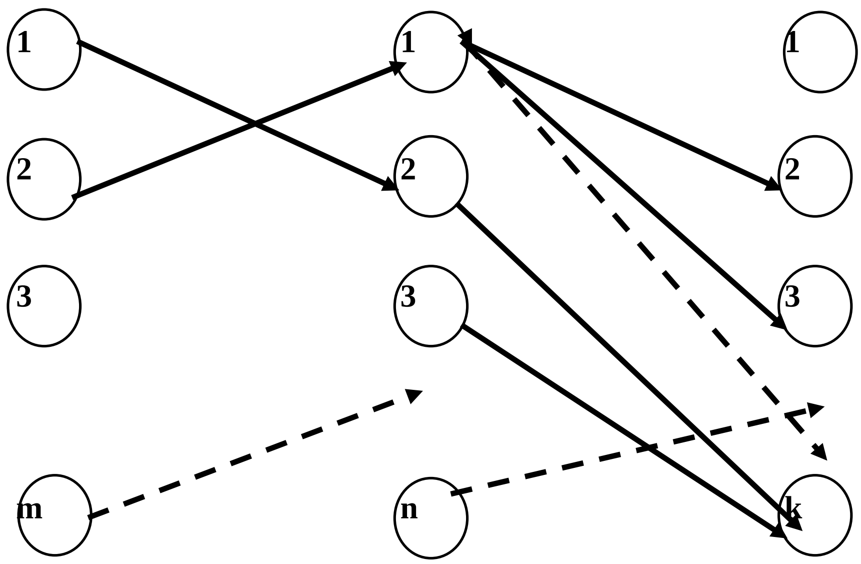
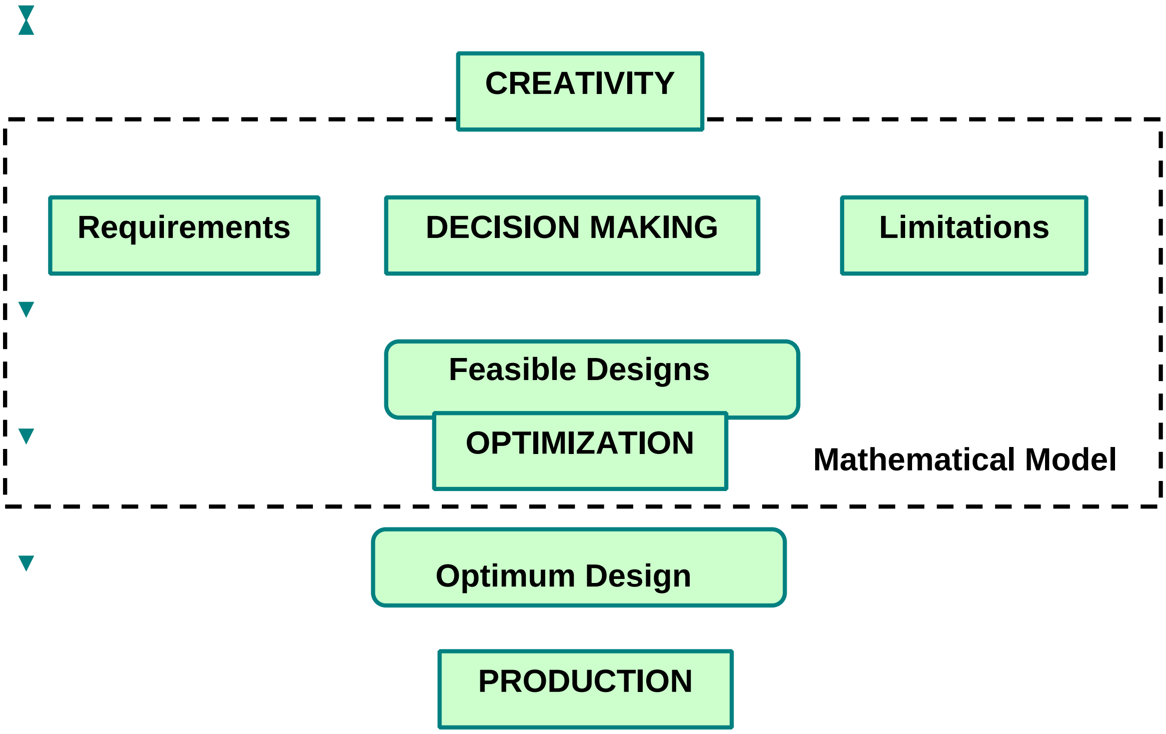
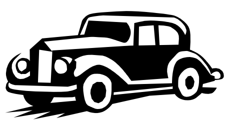
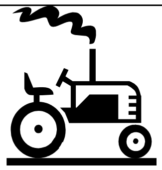


Figure II.11 Successive stages and alternatives in Dynamic Programming.

II.11 COMPUTERS IN OPTIMIZATION

Application of computers in engineering design has fantastically increased the speed of the computations in the analysis of the design problems. Since the designer must always select among the feasible designs, he must solve the problem by considering every possible alternative value. The choice of the final solution is better if the number of alternatives is more. Computers are useful in increasing the number of such alternatives with their high speeds of computations.



	$\{x_1, x_2, x_3, \ldots x_{ncar}\}$ $F_{car} = F_{car}(x_1, x_2, x_3, \ldots x_{ncar})$ $g(x_1, x_2, x_3, \ldots x_{ncar}) = 0$ $h(x_1, x_2, x_3, \ldots x_{ncar}) <> 0$
	$\{x_1, x_2, x_3, \ldots x_{ntractor}\}$ $F_{tractor} = F_{tractor}(x_1, x_2, x_3, \ldots x_{ntractor})$ $g(x_1, x_2, x_3, \ldots x_{ntractor}) = 0$ $h(x_1, x_2, x_3, \ldots x_{ntractor}) <> 0$
	$\{x_1, x_2, x_3, \ldots x_{n\ airplane}\}$ $F_{airplane} = F_{airplane}(x_1, x_2, x_3, \ldots x_{n\ airplane})$ $g(x_1, x_2, x_3, \ldots x_{n\ airplane}) = 0$ $h(x_1, x_2, x_3, \ldots x_{n\ airplane}) <> 0$

Formulation of optimization problems

Maximize/Minimize	$\} F = F(x_1, x_2, x_3, \ldots x_n)$
Subject to:	$g_1(x_1, x_2, x_3, \ldots x_n) = 0$ $g_2(x_1, x_2, x_3, \ldots x_n) = 0$ . . . . $g_m(x_1, x_2, x_3, \ldots x_n) = 0$ where $m<n$ $h_1(x_1, x_2, x_3, \ldots x_n) <> 0$ $h_2(x_1, x_2, x_3, \ldots x_n) <> 0$ . . . . $h_p(x_1, x_2, x_3, \ldots x_n) <> 0$ where p has no limit.

Max./Min $\quad F = F(x_1, x_2, x_3, \dots x_n)$

S. t.: $\quad g_i(x_1, x_2, x_3, \dots x_n)=0 \quad i=1,m$ and $m<n$

$\quad h_j(x_1, x_2, x_3, \dots x_n)<>0 \quad j=1,p$ and p has no limit.

Functional/Regional Constraints

Objective/Criteria Function

UNCONSTRAINED OPTIMIZATION

Max./Min. $\quad \} F = F(x1, x2, x3, \dots xn)$

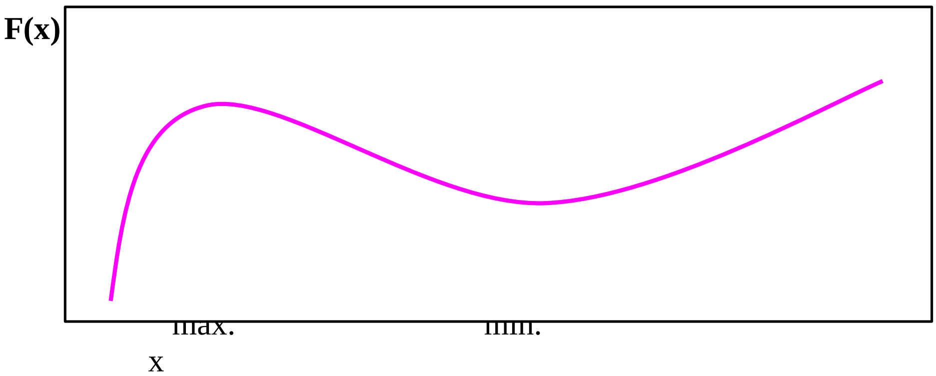
S. t.: $\quad g_i(x_1, x_2, x_3, \dots x_n) =0 \quad i=1, \dots m$ and **m=0**

$\quad h_j(x_1, x_2, x_3, \dots x_n) <>0 \quad j=1, \dots p$ and **p=0**

Max.
Min. $\} F = F(x_1, x_2, x_3, \dots x_n)$

If n=1

F=F(x) and



df/dx = 0 ⇨ optimum x values.

If n > 1 and m=0 and p=0

dF=0 and therefore,

$$\frac{\partial F}{\partial x_1}=0, \quad \frac{\partial F}{\partial x_2}=0, \quad \frac{\partial F}{\partial x_3}=0, \quad \dots\dots\dots\dots \quad \frac{\partial F}{\partial x_n}=0$$

Solve for n variables $\{x_1, x_2, x_3, \dots x_n\}$ and n equations.

CONSTRAINED OPTIMIZATION (Functional constraints)

Max./Min. $F = F(x_1, x_2, x_3, \dots x_n)$

s. t.: $g_i(x_1, x_2, x_3, \dots x_n) = 0$ i=1...m and **0<m<n**

$h_j(x_1, x_2, x_3, \dots x_n) <> 0$ j=1...p and **p=0**

Max/Min $F = F(x_1, x_2, x_3, \dots x_n)$

s. t.: $g_i(x_1, x_2, x_3, \dots x_n) = 0$ i=1…m and **0<m<n**

Method of Lagrange Multipliers

$$dF = \frac{\partial F}{\partial x_1}dx_1 + \frac{\partial F}{\partial x_2}dx_2 + \dots + \frac{\partial F}{\partial x_n}dx_n = 0$$

F dF=0

$$dg_i = \frac{\partial g_i F}{\partial x_1}dx_1 + \frac{\partial g_i}{\partial x_2}dx_2 + \dots + \frac{\partial g_i}{\partial x_n}dx_n = 0$$

g **dg$_i$=0**

\-

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$$dF + \sum_{i=1}^{m} \lambda_i dg_i \qquad \sum_{j=1}^{n}\left(\frac{\partial F}{\partial x_j} + \sum_{i=1}^{m}\lambda_i \frac{\partial g_i}{\partial x_j}\right)dx_j$$

F+λg

=Ł =0 =0 =0

=0

$$d£ = dF + \sum_{i=1}^{m}\lambda_i dg_i = \sum_{j=1}^{n}\frac{\partial}{\partial x_j}\left(F + \sum_{i=1}^{m}\lambda_i g_i\right)dx_j = 0$$

Lagrange Function

$£ = F + 0$ numerical values are same. Therefore $£_{opt} = F_{opt}$

$$£ = £(x_1, x_2, x_3, \dots, x_n, \lambda_1, \lambda_2, \lambda_3, \dots \lambda_m)$$

$$\frac{\partial £}{\partial x_1} = 0, \quad \frac{\partial £}{\partial x_2} = 0, \quad \frac{\partial £}{\partial x_3} = 0, \quad \dots \quad \frac{\partial £}{\partial x_n} = 0, \quad \frac{\partial £}{\partial \lambda_1} = 0, \quad \frac{\partial £}{\partial \lambda_2} = 0, \quad \dots \quad \frac{\partial £}{\partial \lambda_m} = 0,$$

Method of Parameter Elimination

Max./Min $F = F(x_1, x_2, x_3, \ldots x_n)$

S. t.: $g_i(x_1, x_2, x_3, \ldots x_n)=0$ $\quad i=1,m$ and $\mathbf{0<m<n}$

Eliminate g_i

$g_i(x_1, x_2, x_3, \ldots x_{r-1}, \mathbf{x_r}, x_{r+1}, \ldots x_n)=0$

$\mathbf{x_r}=r(x_1, x_2, x_3, \ldots x_{r-1}, x_{r+1}, \ldots x_n)$

$F=F(x_1, x_2, x_3, \ldots x_{r-1}, \mathbf{x_r}, x_{r+1}, \ldots x_n)$

$F'=F'(x_1, x_2, x_3, \ldots x_{r-1}, x_{r+1}, \ldots x_n)$

n is reduced by 1, one of g_i is eliminated.

Use all g_i I=1…m to eliminate m x variables.

$F=F(X_1, X_2, X_3, \ldots X_N)$

Where N=n-m

Therefore, we have reduced the constrained problem to an unconstrained problem. Hence.

$$\frac{\partial F}{\partial X_1}=0 \quad \frac{\partial F}{\partial X_2}=0 \quad \ldots \quad \frac{\partial F}{\partial X_N}=0$$

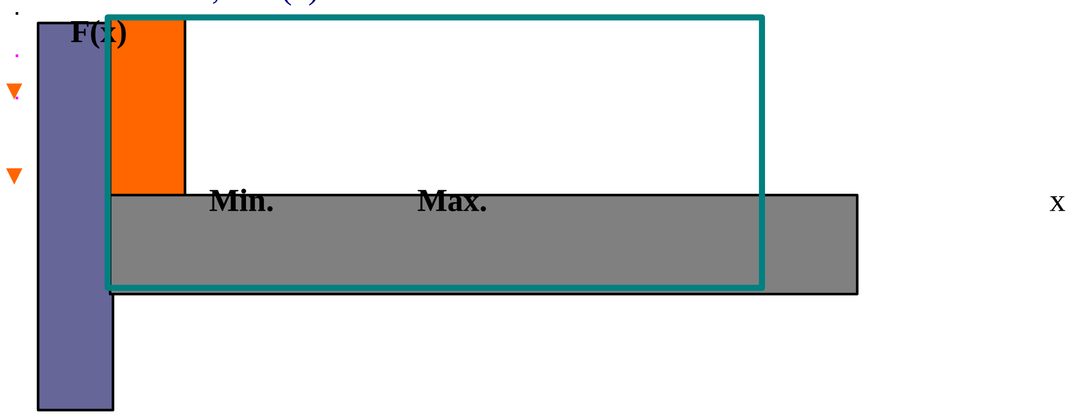
CONSTRAINED OPTIMIZATION (Regional constraints)

Max./Min $F = F(x_1, x_2, x_3, \ldots x_n)$

s.t.: $g_i(x_1, x_2, x_3, \ldots x_n)=0$ $\quad i=1\ldots m$ and $\mathbf{m=0}$

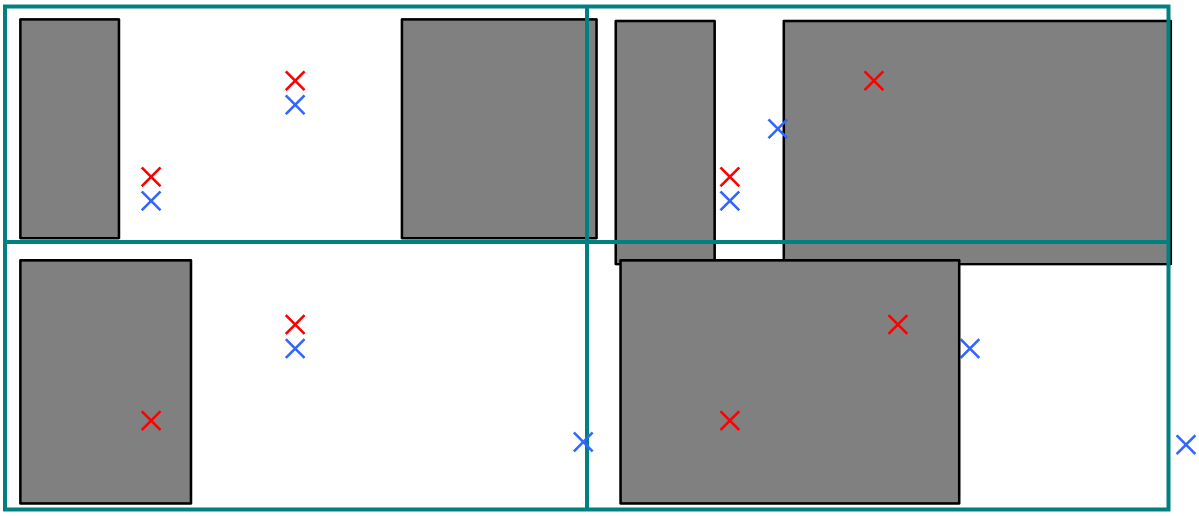
$h_j(x_1, x_2, x_3, \ldots x_n) <> 0$ $\quad j=1\ldots p$ and $\mathbf{p>0}$

Consider n=1, F=F(x)

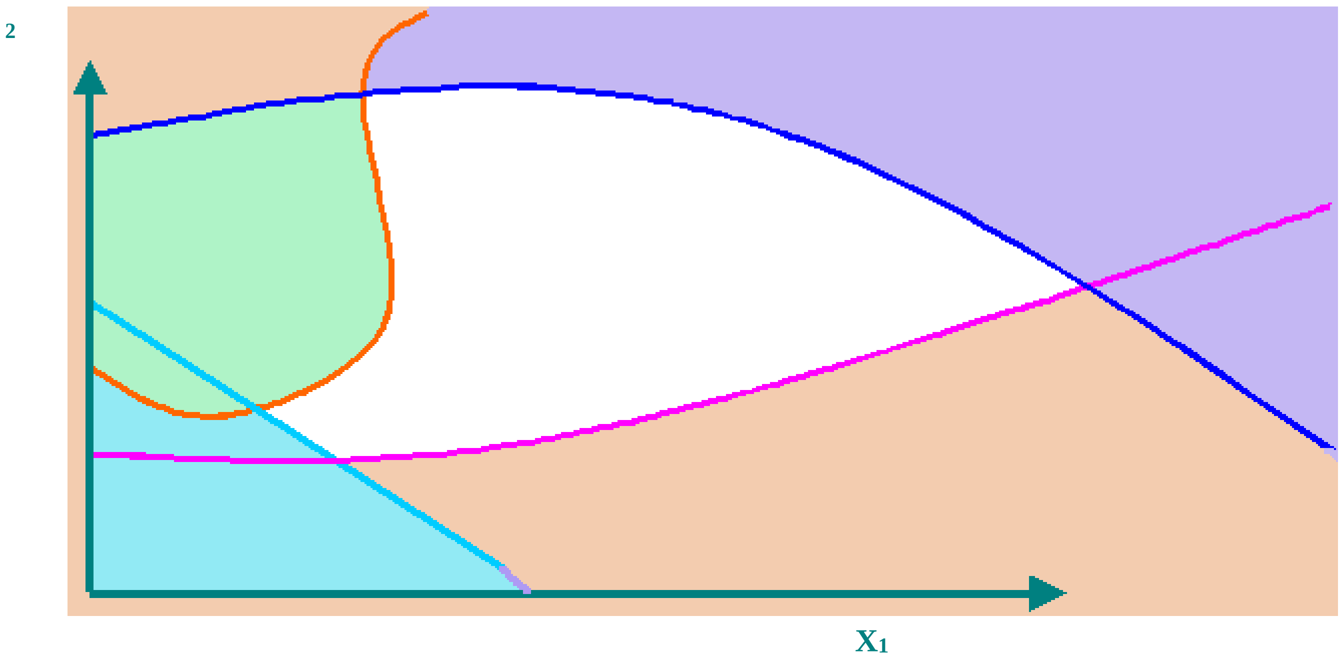


Non-negativity conditions

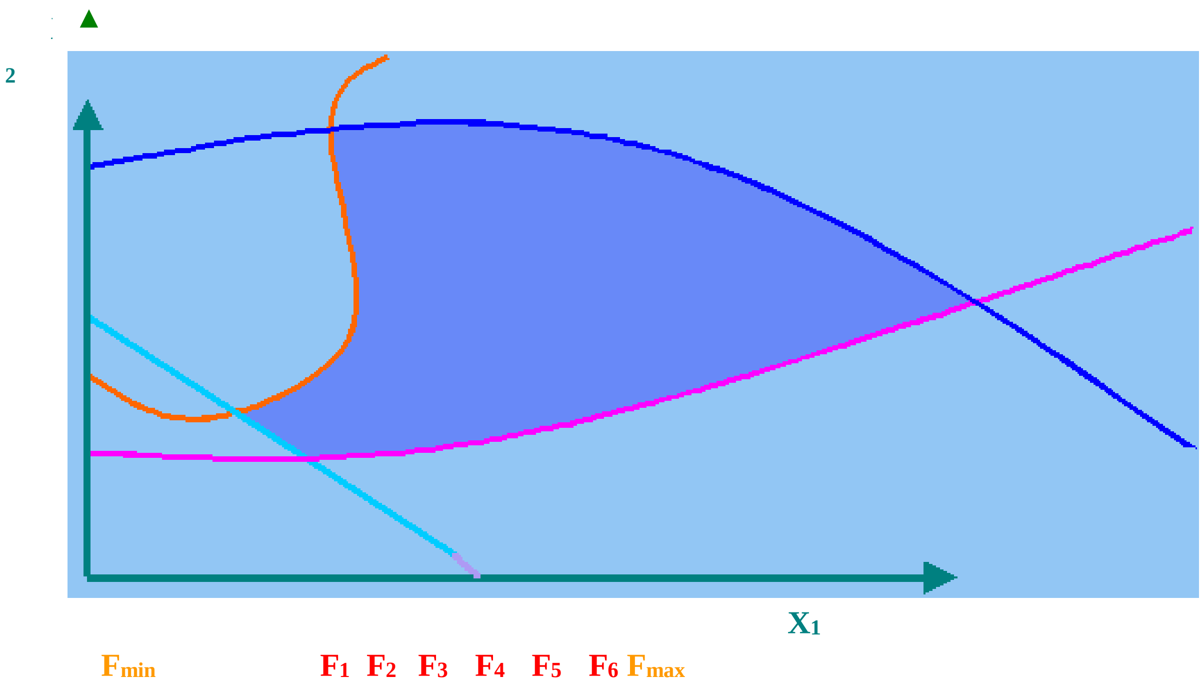
Regional constraint (**ΦD>5 cm**)



Mathematical optimum and feasible optimum points are different concepts.



Feasible Region



$F_{min} < F_1 < F_2 < F_3 < F_4 < F_5 < F_6 < F_{max}$

Any Machine Element

1- Functional requirement parameters
2- Desirable/undesirable effects
3- Uniqueness

Design Parameters

1- Functional requirement parameters
2- Desirable/Undesirable effect parameters
3- Geometrical parameters
4- Material parameters

Procedure

Step 1- Draw a free hand sketch. Select independent design parameters to define the artifact uniquely.

Step 2- Decide on the most significant criterion and write $F=F(x_1, x_2, ...,x_n)$

Step 3- Write all related equations $g(x_1, x_2, ...,x_n)=0$.

Step 4- Write limit relations. $h(x_1, x_2, ...,x_n)><0$.

Step 5- Eliminate common parameters between g and h.

<u>Rules</u>: Eliminate unlimited parameters

Do not eliminate material parameters

Step 6- Draw rough sketches for F vs x_i for all x_i except material parameters.

Step 7- Apply mathematical optimization techniques to find optimum solution.

Step 8- Determine material selection factor and select the optimum material.

Step 9- Determine the optimum values of the eliminated parameters.

List of significant criteria

A) List the criteria in the order of significance

F_x ➡ Objective Function / Primary Design Criterion

F_y

F_u

$\longrightarrow$ $g(x_1, x_2, ...,x_n)=0$ or $h(x_1, x_2, ...,x_n) >< 0$.

F_z

Solve for the optimum values as formulated here. If some of the parameters are not determined, drop F_x, move F_v up and re-solve the optimization problem. Repeat this procedure until all of the design parameters are determined.

B) Define an equivalent criterion function.

$F=\psi_1 F_1 + \psi_2 F_2 + \psi_3 F_3 + \ldots \psi_k F_k$

Where $\psi_1, \psi_2, \psi_3, \ldots \psi_k$ are weighing factors.

Limits

- Loose limits
- Rigid limits

In this text, the most common optimization methods in mechanical engineering applications will be considered very briefly. Theoretical basis of the methods and their details will not be studied. It should be remembered that the mathematical approaches given here are adapted for mechanical engineering design applications. Thus, the techniques discussed in this text may have differences from the purely mathematical applications, advanced mathematical models must be studied. Literature for further reading is given at the end of the text.

It is possible to classify the optimization methods in several ways. Constrained and Unconstrained Optimization is one way, Numerical or Analytical Optimization is another way. For such a general classification, mathematical backgrounds should be considered in detail. Since our aim in this text is mechanical engineering design applications only, we shall consider most applicable methods under different headings, even if their mathematical basis is the same.

T.3 Apply method of differentiation

Q.4 Is the criterion function differentiable?

Q.5 Is the method of calculus of variations applicable?

T.4 Apply calculus of variations techniques

R Reformulate the problem.

T.5 Apply engineering optimization approaches.

Figure 2.2 Arbitrary forms of a one -variable function (F(x)

II.5 METHOD of LINEAR PROGRAMMING

It was shown that any optimization problem can be formulated in three groups of equations.

Max./Min. $F=F(x_1,x_2,......, x_n)$

s.t. $g_i(x_1,x_2,.........., x_n) = 0$ $i = 1, m$ $m<n$

$h_i(x_1, x_2,........., x_n)\ \varepsilon\ 0$ $i = 1, p$ p has no limit.

There is a vast group of problems where the above relations are all linear. Thus, the general form of the optimization problem is in the following form:

Max.Min. $F=f_1x_1+f_2x_2+f_3x_3+..........+f_nx_n$

s.t. $g_{i1}x_1+g_{i2}x_2+...........+g_{in}x_n+c_i = 0$

$i = 1, m$ $m<n$

$h_{i1}x_1+h_{i2}x_2+............+ h_{in}x_n+k_i\varepsilon 0$

$i = 1, p$ p has no limit.

Where C_i and K_i are plain numerical constraints.

Further the problem can be so arranged that all the design parameters can have positive values only.

$x_1\ \varepsilon\ 0,$ $x_2\ \varepsilon\ 0, ...x_n\ \varepsilon\ 0$

This is called non-negativity condition.

The above problem can be re-formulated in the matrix form. The criterion function is a product of two matrices. That is;

$F = [\ f\ ]^T [\ x\ ]$

Where $[\ f\ ]^T$ is the transpose of $[\ f\ ]$ and;

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and non-negativity conditions in matrix form are:

[x] ε [0]

Considering the above matrix equations, the optimization problem can be written in terms of four matrix relations.

Max. Or Min. F = $[f]^T [x]$

s.t. [G][x] = [C]

[H][x] ε [K]

[x] ε [0]

Any optimization problem which is formulated in this form can be solved by a method which is called Linear Programming. It is possible to show the method of solution by matrix algebra, but it is beyond the scope of this text. Two dimensional problems can be solved by purely graphical means, and it is very useful. If the number of parameters is more than 2, there are some methods (most applicable one is Simplex Tableaux) to obtain the optimum point. Although hand calculations are possible it is time consuming and prone to make calculation errors. Instead, computer techniques are preferred. Almost all companies provide ready-to-use Linear Programming packages supplied with their computers. The only work to be done by the user is to supply the numerical coefficients; in other words, the matrices [f], [G] and [H] must be given as the input data. The optimum point (if exists) and some information is given as output by the computer.

Some optimization problems cannot be readily put in a format suitable for Linear Programming. The most common headache for mechanical design applications is the non-negativity conditions. Generally, we have;

M δ x_r δ + M

where x_r is any design parameter and M is a positive number, including □. It is possible to define two new parameters x_r' and x_r'' such that;

$$x_r = x_r^1 - x_r^{11}$$

with the condition.

$$x_r^1 \geq 0, \qquad x_r^{11} \geq 0$$

Therefore, x_r is replaced by $x_r^1 - x_r^{11}$, the linearity of the relations are still maintained and non-negativity conditions are satisfied.

If the criterion function or constraints are not in linear form, we can either approximate them as linear equations in certain ranges (see Fig. II.5) or apply some mathematical tricks. To illustrate this suppose we have a function as:

$$F = x_1^a 1 x_2^a 2 x_3^a 3 ....x_n^a n$$

where a_1, a_2, ...a_n are constants. This is an exponential equation and cannot be used in Linear Programming. If we take logarithm of both sides, we obtain a linear equation in terms of logarithms of parameters.

$$\ell nF = a_1 \ell n x_1 + a_2 \ell n x_2 + ... + a_n \ell n x_n$$

Defining new parameters as;

$$F^1 = \ell n F$$

$$x_1^1 = \ell n x_1, \quad x_2^1 = \ell n x_2, \ldots\ldots\ldots x_n^1 = \ell n x_n$$

we have

$$F^1 = a_1 x_1^1 + a_2^1 x_2^1 + \ldots\ldots\ldots a_n x_n^1$$

This is a linear equation, and the method can be applied without any difficulty. Some other mathematical tricks, depending on the user's ingenuity can be found and applied successfully.

It was stated that two-dimensional Linear Programming problems can be solved by graphical methods. This will be illustrated now. Consider a two-parameter problem in its most general form.

Max. Min. $F = f_1 x_1 + f_2 x_2$

$$g_1 x_1 + g_2 x_2 + C = 0$$
$$h_{11} x_1 + h_{12} x_2 + K_1 \geq 0$$
$$h_{21} x_1 + h_{22} x_2 + K_2 \geq 0$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$h_{p1} x_1 + h_{p2} x_2 + K_p \geq 0$$

s.t. $x_1 \geq 0, x_2 \geq 0$

All these relations can be shown on a 2-dimensional plot. Firstly, non-negativity conditions exclude all the quadrants except the first. Consider criterion function F in the first quadrant. Since f_1 and f_2 are known numerically, for any arbitrary value of F, a straight line can be drawn in x_1-x_2 plane (Fig, II.6). F_1, F_2, ...are such lines and they are naturally parallel. The value of F increases in the direction of arrow depending on the signs of f_1 and f_2. That is;

Either $F_1>F_2>F_3>F4$.....or $F_1<F_2<F_3<F_4$ is true.

Thus, the optimum (Maximum or Minimum) point can be reached by moving in the direction (or against it) of arrow. How far we will move is determined by the constraints.

The functional constraint defines a line in x_1-x_2 plane and if it is satisfied by the design parameters, it defines a feasible region along the line segment AB only. Any point on this line is a feasible point and the optimum point is one of these (Fig.II.7).

Consider the regional constraints in x_1-x_2 plane. Let us take the first,

$$h_{11} x_1 + h_{12} x_2 + K_1 \leq 0$$

Figure II.5

Figure II.6

Figure II.7

Figure II.8

Depending on numerical values and signs of h_{11}, h_{12} and K, the inequality relation divides the first quadrant into two regions (Fig. II.8). The points in the hatched region do not satisfy the inequality therefore they are not feasible points. Any point in the feasible region may be an optimum point. When the regional constraints are plotted on the same figure, we obtain the common feasible region defined by all of the regional constraints (Fig. II.9). Further imposing the functional constraint on this figure (Fig. II.10), we have the final feasible region as a line segment in x_1-x_2 plane. Usually, such problems do not have functional constraints, thus similar regions as shown in Fig. II.9 are obtained.

Figure II.9

Figure II.10

To find the optimum point, it is sufficient to draw the parallel lines representing the criterion function. The maximum or minimum point will be on one of the corners of the feasible region.

Applications of Linear Programming problems can be summarized in 4 groups.

1. Alloying: Several elements with different costs and properties are considered and Linear Programming is applied to find minimum cost with the given constraints. Metallurgical alloying, liquid blending, ore combinations and cattle feed mixing are examples.
2. Job Allocation: The machines (capacity and time) labor, and raw materials are arranged such that the total profit from production of several items is maximized.
3. Production Scheduling: The production rates of several products are determined by considering Supply, Demand, Inventory and Storage facilities.

4. Transportation: The best routes and schedules are determined for all sorts of vehicles.

II.6 AN APROACH TO SOLUTIONS OF OPTIMIZATION PROBLEMS FOR ENGINEERING APPLICATIONS

An optimization method, applied particularly in mechanical engineering design, is described by Johnson in his several books and papers (14, 15). It is a clearly described procedure for optimization problems rather than being a mathematical optimization method. The steps of optimization are clearly defined and confusion in many variable problems are reduced by a systematic approach. Several interesting examples are solved by the described procedure. We think that the procedure given by Johnson can be applied to any optimization problem and the mathematical methods described in the previous sections can be used within the procedure. In the next sections, the procedure will be described with some small changes. But before this, there are several concepts which must be explained clearly.

Any machine or machine element has generally three basic properties.

4. Functional Requirement Properties.
5. Desirable or Undesirable Effects.
6. Uniqueness.

Now, we shall study these properties:

4. Functional Requirement Properties: Any machine or machine element is designed for a certain purpose. It must perform its functions in this predetermined way. Consider a spring, which is designed to exert a force F. Then the force F is a functional requirement. It is a property that we expect from the spring. Similarly, a shaft must transmit torque, a gearbox must transmit the power at a certain input and output speed, a cam must give a predetermined displacement to its follower, and finally a structural number (leg of a table in its simplest form) must carry a certain weight. It is clear that the functional requirements must be satisfied in a predetermined way, as predicted by the designer.

5. Desirable and Undesirable Effects: In designing the machines and its elements to satisfy the functional requirements, there are some properties which are unavoidable for a real physical system. For example, a spring cannot be designed with an imaginary material and since all of the real materials have a certain upper limit for the stresses, then the stress is an unavoidable property. Similarly, any real material has a weight under the gravitational field and it is again unavoidable. Deflection, space occupancy, vibration and natural frequency are other properties for any simple helical spring. Some of these properties are desirable and some are undesirable depending on the application. For example, weight in some applications can be desired whereas in some others it is not desired. Other examples for desirable and undesirable effects are surface quality, machinability, length, diameter, appearance, comfortability stability, volume, cost, production time etc. All of these effects, if significant, must be considered in any optimization problem.

6. Uniqueness: Any machine or machine element can be defined completely if all the dimensional parameters and the materials parameters are specified. This statement can be put in reverse order; any machine or machine element can be specified completely

once all the material parameters and geometrical parameters are known. The geometrical parameters are length, diameter, width, height, tolerances etc., and the material parameters are Elasticity modulus, Yield Point, Weight density, Unit Cost, Poisson's ratio etc.

In any design problem statement, the above properties are either specified numerically or their allowable limits are given. Thus, a diameter can be required to lie between, say, 5 mm and 15 mm or a spring force can be stated to have values 50 kg ± 10 kg. Λ natural frequency can have any value between 0 and □ in its widest range. Usually, all the properties and their corresponding parameters have lower and upper limits. If these lower and upper limits are required, it is called a RIGID LIMIT, if slight changes on these limiting values are permissible, then these are called LOOSE LIMITS. Loose limits are useful when it is possible to obtain better solutions for the design problems with slight changes in the limiting values. Apparently, these "Slight changes" should not make the solution impossible or impractical.

During the mathematical formulation of the problem, the above stated properties are expressed in terms of four groups of parameters.

5. Functional Requirement parameters,
6. Desirable and Undesirable effect parameters,
7. Geometrical parameters,
8. Material parameters.

Functional Requirement parameters include the mathematical equivalents of the functional requirement properties. They are usually external to the machine or machine element. Mostly they express the relation between the neighborhood machines. (Torque of gear is transmitted from a shaft, and shaft receives the power from an electric motor). Thus, before starting a design, the functional requirement parameters must be known and determined due to the external effects. Sometimes the resulting values of these parameters are defined as the effect of the design to the design to the surroundings. In both cases, it is considered as the overall reaction to the next machine or element (system).

1. Desirable and Undesirable Effect parameters are determined terms of design parameters during the design process. They do not affect the design of the other machines or elements directly. Only a cumulative effect on the most general system is possible. For example, the total cost of a machine is the sum of the cost of each machine element.
2. Geometrical parameters are usually independent of each other, but they determine the previous two groups of parameters. If they are limited, the geometrical parameters are dependent on each other. There may be also limiting values on the geometrical parameters. Usually, the designer finds great freedom in changing the geometrical parameters independently. But he must be careful not to violate the limits. The geometrical parameters are mostly continuous variables like diameter or length of a shaft to be turned on a lathe. But sometimes they have stepwise variation. The standardized machine elements (Roller bearings, bolts, screws, gears (partially) and others). Have some preferred dimensions and the designer must make his choice among these values.
3. Material parameters are described in discussing material properties. Their common property is the interdependence of all the material parameters. If some of the

material parameters are determined (say, yield point) the number of alternative materials is limited. In any design application, the number of available materials is limited and not more than 10 or a close figure. If the material name is known, then all of the material parameters are known. Specifying C-1040 with its commercial name is equivalent to specifying all its related parameters.

Most of the parameters in these four groups must remain within some practical limiting is called a feasible parameter set. One of these sets is the optimum design. The optimization is based on one of the functional requirements and/or desirable and undesirable effects.

There are three groups of mathematical relations in any optimization problem as was discussed before. The names given previously were adapted from mathematical theories. We shall now introduce and use names which are more proper for engineering design.

The Primary Design Equation (PDE) corresponds to the criterion function. It is the most significant functional requirement, Desirable effect, or Undesirable effect. The choice of maximization and minimization depends on the problem. The most common applications are given below.

Minimize: Cost, Weight, Volume, Deflection, Natural Frequency, Length, Speed, Instability, Force, Area, Stress, ...

Maximize: Power Transmission Capability, Energy Storage, Speed, Natural Frequency, Weight, Length, Stability, Force, Area, Safety Factor, ...

The designer must be careful that some negative properties like cost must never be maximized and positive properties like efficiency, power transmission capability must never be minimized.

Besides the nature of the property to be optimized, the application also determines the choice. Consider a shaft, exactly similar in two different applications like Stone Crashing machine and Aircraft. The criterion of shaft design for the Aircraft will be “minimum weight”. The weight is not important at all in the first case and perhaps torque transmission capability will be the design criterion.

Subsidiary Design Equations (SDE) are all of the equality relations other than the Primary Design Equation. These were called previously as functional constraints.

Limit Equations are the regional constraints of mathematical methods.

All the equations should be arranged such that the parameters stated above will appear as independent parameter groups. This is not always possible, and some mathematical tricks (like defining new parameters) is applied in such cases. These are illustrated in the examples to be given later.

The procedure of optimization has 9 steps:

Step 1: Draw a free hand sketch of the system and show all of the related parameters. Select the independent parameters which will uniquely define the geometry. If a choice exists, select parameters which are either specified as functional requirements or limited to permissible values.

Step 2: Decide on the most significant criterion for optimization and write the mathematical expression. This is the Primary design equation (PDE). If possible, write this equation in terms of parameter groups.

Step 3: Write the equations for all other significant functional requirements and desirable and undesirable effects. These equations are Subsidiary Design Equations (SDE).

Step 4: Write the mathematical limit equations for all the parameters. Indicate rigid and loose limits.

Step 5: Eliminate a common design parameter from the Primary Design Equation for each Subsidiary Design Equation. Re-write the remaining Subsidiary Design Equations and Limit equations in terms of the remaining design parameters. If choice exists eliminate unlimited and unspecified parameters. Do not eliminate material parameters since they are dependent on each other and have limited ranges. Dimensional parameters limited within finite ranges should not be eliminated, if it is not necessary. At the end of this step, the original design equation is written in terms of limited parameters and geometrical parameters. This equation is called Develop Primary Equation (DPDE).

Step 6: From the Develop PDE, obtain variation of the design criterion w.r.t. each of the parameters (except material parameters). Draw rough sketches indicating the general trend of the criterion within the feasible range of the parameters.

Step 7: Considering the variations determined in the previous step, obtain optimum design quantities. If it is necessary, apply the mathematical optimization techniques. The set of optimum parameters defined at this step must define a unique system with the design criterion as optimized.

Step 8: The only remaining parameters are material parameters, and they can be independently isolated from the other parameters before the 7th step. Select the optimum material by considering the Material Selection Factor and available materials for the design problem. Remember that the optimum material must be one of the available materials and material parameters are dependent on each other.

Step 9: Determine optimum values for the eliminated parameters by using the known optimum parameters.

II.7 METHOD of OPTIMUM SHAPE DESIGN

Sometimes it is required to determine the optimum shape of machine element with a particular criterion. Examples of this sort are the shape which gives minimum circumferential length for a fixed area, or the shape of a connecting arm for maximum strength. Solution of such problems is not very simple and usually requires the application of advanced mathematical techniques. Sometimes numerical methods are applied.

Optimization problems for optimum shape design are generally solved by two methods.

i) Direct methods
ii) Method of Calculus of Variations

In this section, the direct methods will be studied. The value of criterion is not usually unlimited, and the optimum point is obtained at either upper or lower limit. For example, cost is always required to be minimized. Thus, the ideal optimum solution is zero which is never possible. There must be a lowest limit of the cost which is defined by some factors necessary for the physical realization of the system. Strength is such an important factor. In other words, to have the necessary minimum strength for a, say, beam, a certain amount of material must be used and that is equivalent to the cost. If we can design the system such that the effective factors are kept at their limiting values for all of the possible conditions, then we obtain the "best solution" and it is an optimum design. Mathematically some of the regional constraints are converted to functional constraints by considering equality sign instead of inequalities. This cannot be done for all regional constraints since in that case, no feasible solution can be obtained. Consider an optimization problem, mathematically stated in its proper form as it is discussed before.

$$Opt.F = F(x_1, x_2, x_3, ...x_n)$$

$$s.t.g_i(x_1, x_2, x_3, ..., x_n) = 0$$ i = 1,..m, m<n

$$h_i \ (x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n) = 0 h_i \ (x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n) = 0$$

i = 1, p, p has no limit

In usual applications of this sort, m=0 and p=1 for a practical solution. The boundary of the system defined by h^1 function is the only feasible solution and it is optimum.

II.8 OPTIMUM DESIGN BY THE METHOD OF CALCULUS OF VARIATIONS

There is a group of optimization problems which have the criterion function in integral equation form.

$$I = \int_{x_1}^{x_2} F(x, y, y^1) dx$$

where I is the criterion function, x is the design parameter, y=y(x) is a mathematical function and y^1=dy/dx. The integrand function F is known explicitly, and it is required to determine y(x) which optimizes the integral value I. By mathematical methods it is possible to show that the optimum point is obtained when the Euler-Lagrange condition is satisfied.

$$\frac{aF}{ay} - \frac{d}{dx}\frac{aF}{a_y 1} = 0$$

Knowing F explicitly, one obtains a differential equation which can be solved to determine the optimum function y(x).

II.9 OPTIMIZATION BY NUMERICAL METHODS

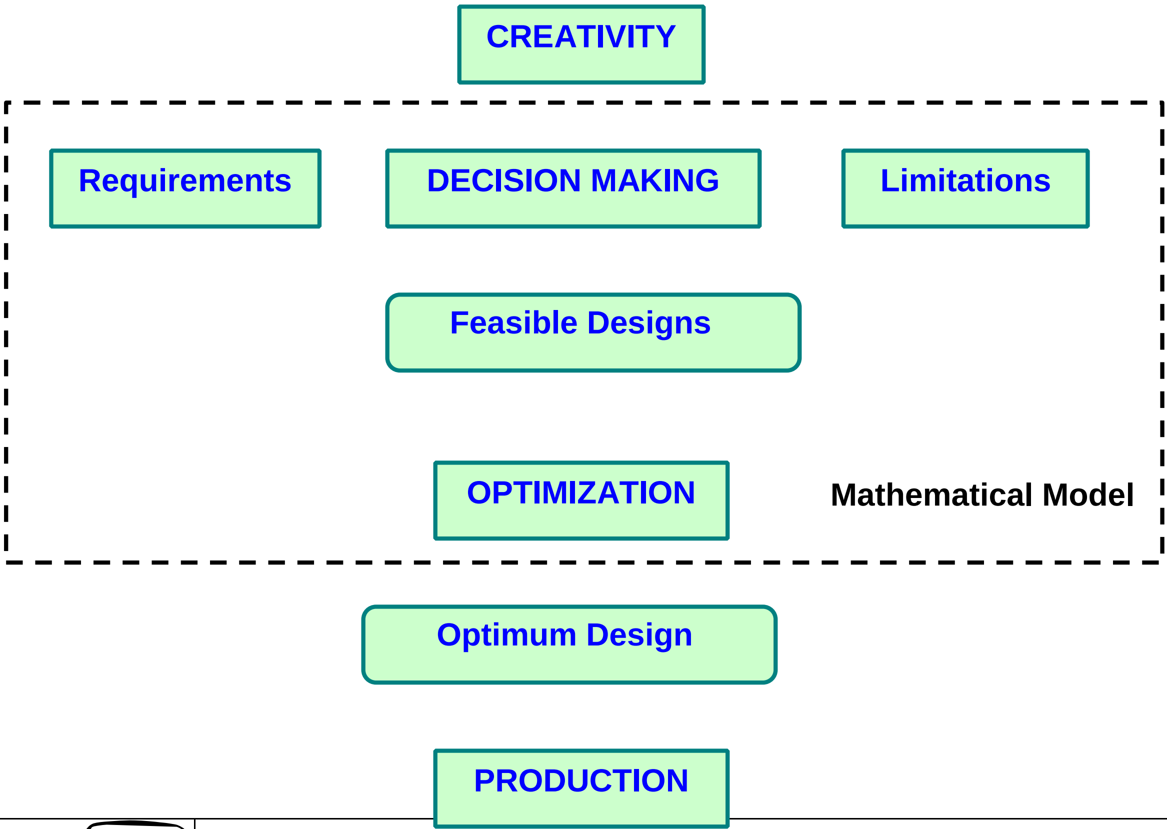
When the number of parameters is large, it is usually difficult to apply manual computation. Numerical methods in such cases help us to find the optimum solutions. It is a broad area of study and beyond the scope of this text. Several methods, like. Exhaustive search, area elimination, Fibonacci search, Golden section method, Grid search and Gradient search are discussed in several other books and some of the important references are given in the reference list.

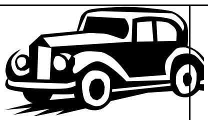
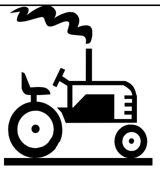
II.10 METHOD of DYNAMIC PROGRAMMING

Dynamic programming is developed to solve a special class of problems where multi-stage decisions are required. For example, consider a problem where we start from the first stage and reach to Nth stage (Fig. II.11). There may be several alternative points to start on at the first stage and there may be several alternative end points at the final stage. Further, the intermediate stages may have many-different alternatives. Since in each stage, we must select a single alternative, then the successive selection of these alternatives will change the value of the criterion function. The types of problems solved by dynamic programming are Network problems, resource Allocation and Reliability problems or similar cases.

II.11 COMPUTERS IN OPTIMIZATION

Application of computers in engineering design has fantastically increased the speed of the computations in the analysis of the design problems. Since the designer must always select among the feasible designs, he must solve the problem under considering for every possible alternative value. The choice of the final solution is better if the number of alternatives is more. The computers are useful in increasing the number of such alternatives with their high speeds of computations.



	$\{x_1, x_2, x_3, \dots x_{ncar}\}$ $F_{car} = F_{car}(x_1, x_2, x_3, \dots x_{ncar})$ $g(x_1, x_2, x_3, \dots x_{ncar}) = 0$ $h(x_1, x_2, x_3, \dots x_{ncar}) <> 0$
	$\{x_1, x_2, x_3, \dots x_{ntractor}\}$ $F_{tractor} = F_{tractor}(x_1, x_2, x_3, \dots x_{ntractor})$ $g(x_1, x_2, x_3, \dots x_{ntractor}) = 0$ $h(x_1, x_2, x_3, \dots x_{ntractor}) <> 0$

$\{x_1, x_2, x_3, \ldots x_{n\ airplane}\}$ $F_{airplane} = F_{airplane}(x_1, x_2, x_3, \ldots x_{n\ airplane})$

$g(x_1, x_2, x_3, \ldots x_{n\ airplane}) = 0$ $h(x_1, x_2, x_3, \ldots x_{n\ airplane}) <> 0$

Formulation of optimization problems

Maximize Minimize	} $F = F(x_1, x_2, x_3, \ldots, x_n)$
Subject to:	$g_1(x_1, x_2, x_3, \ldots, x_n) = 0$ $g_2(x_1, x_2, x_3, \ldots, x_n) = 0$ . . . . $g_m(x_1, x_2, x_3, \ldots, x_n) = 0$ where $m<n$ $h_1(x_1, x_2, x_3, \ldots, x_n) <> 0$ $h_2(x_1, x_2, x_3, \ldots, x_n) <> 0$ . . . . $h_p(x_1, x_2, x_3, \ldots, x_n) <> 0$ where p has no limit.

Max./Min $F = F(x_1, x_2, x_3, \ldots, x_n)$

S. t.: $g_i(x_1, x_2, x_3, \ldots, x_n)=0$ i=1,m and m<n
$h_j(x_1, x_2, x_3, \ldots, x_n)<>0$ j=1,p and p has no limit.

▲ Functional/Regional Constraints
▲ Objective/Criteria Function

II.13 UNCONSTRAINED OPTIMIZATION

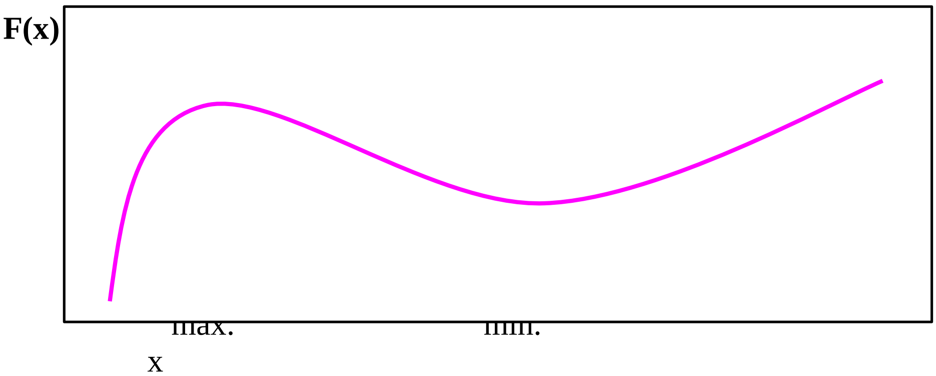
Max./Min. } $F = F(x1, x2, x3, \ldots, xn)$

S. t.: $g_i(x_1, x_2, x_3, \ldots, x_n) = 0$ i=1, ... m and **m=0**
$h_j(x_1, x_2, x_3, \ldots, x_n) <> 0$ j=1, … p and **p=0**

Max.
Min. } $F = F(x_1, x_2, x_3, \ldots, x_n)$

If n=1

F=F(x) and



df/dx = 0 $\Rightarrow$ optimum x values.

If n > 1 and m=0 and p=0

dF=0 and therefore,

$$\frac{\partial F}{\partial x_1}=0,\quad \frac{\partial F}{\partial x_2}=0,\quad \frac{\partial F}{\partial x_3}=0,\quad ....\quad \frac{\partial F}{\partial x_n}=0$$

Solve for n variables {x_1, x_2, x_3, …. x_n} and n equations.

II.14 CONSTRAINED OPTIMIZATION (Functional constraints)

Max./Min. F = F (x_1, x_2, x_3, …. x_n)

s. t.: $g_i(x_1, x_2, x_3, ... x_n) = 0$ i=1...m and **0<m<n**

$h_j(x_1, x_2, x_3, ... x_n) <> 0$ j=1...p and **p=0**

Max/Min F = F (x_1, x_2, x_3, …. x_n)

s. t.: $g_i(x_1, x_2, x_3, ... x_n) = 0$ i=1…m and **0<m<n**

Method of Lagrange Multipliers

F $\Rightarrow$ dF=0 $$dF=\frac{\partial F}{\partial x_1}dx_1+\frac{\partial F}{\partial x_2}dx_2+.....+\frac{\partial F}{\partial x_n}dx_n=0$$

g $\Rightarrow$ **dg_i=0** $$dg_i=\frac{\partial g_i F}{\partial x_1}dx_1+\frac{\partial g_i}{\partial x_2}dx_2+.....+\frac{\partial g_i}{\partial x_n}dx_n=0$$

\-

\+

F+λg $\Rightarrow$ $dF+\sum_{i=1}^{m}\lambda_i dg_i$ $\Rightarrow$ $\sum_{j=1}^{n}\left(\frac{\partial F}{\partial x_j}+\sum_{i=1}^{m}\lambda_i\frac{\partial g_i}{\partial x_j}\right)dx_j$

=Ł =0 =0 =0

=0

$$d£=dF+\sum_{i=1}^{m}\lambda_i dg_i=\sum_{j=1}^{n}\frac{\partial}{\partial x_j}\left(F+\sum_{i=1}^{m}\lambda_i g_i\right)dx_j=0$$

Lagrange Function

$£=F+0$ numerical values are same. Therefore $£_{opt}=F_{opt}$

$$£=£(x_1,x_2,x_3,.....,x_n,\lambda_1,\lambda_2,\lambda_3,...\lambda_m)$$

$$\frac{\partial £}{\partial x_1}=0,\ \frac{\partial £}{\partial x_2}=0,\ \frac{\partial £}{\partial x_3}=0,\ ...\ \frac{\partial £}{\partial x_n}=0,\ \frac{\partial £}{\partial \lambda_1}=0,\ \frac{\partial £}{\partial \lambda_2}=0,\ ...\ \frac{\partial £}{\partial \lambda_m}=0,$$

Method of Parameter Elimination

Max./Min $F = F(x_1, x_2, x_3, \ldots x_n)$

S. t.: $g_i(x_1, x_2, x_3, \ldots x_n)=0$ $i=1,m$ and $\mathbf{0<m<n}$

Eliminate g_i

$g_i(x_1, x_2, x_3, \ldots x_{r-1}, \mathbf{x_r}, x_{r+1}, \ldots x_n)=0$

$\mathbf{x_r}=r(x_1, x_2, x_3, \ldots x_{r-1}, x_{r+1}, \ldots x_n)$

$F=F(x_1, x_2, x_3, \ldots x_{r-1}, \mathbf{x_r}, x_{r+1}, \ldots x_n)$

$F'=F'(x_1, x_2, x_3, \ldots x_{r-1}, x_{r+1}, \ldots x_n)$

n is reduced by 1, one of g_i is eliminated.

Use all g_i I=1…m to eliminate m x variables.

$F=F(X_1, X_2, X_3, \ldots X_N)$

Where N=n-m

Therefore, we have reduced the constrained problem to an unconstrained problem. Hence.

$$\frac{\partial F}{\partial X_1}=0 \quad \frac{\partial F}{\partial X_2}=0 \quad \ldots \quad \frac{\partial F}{\partial X_N}=0$$

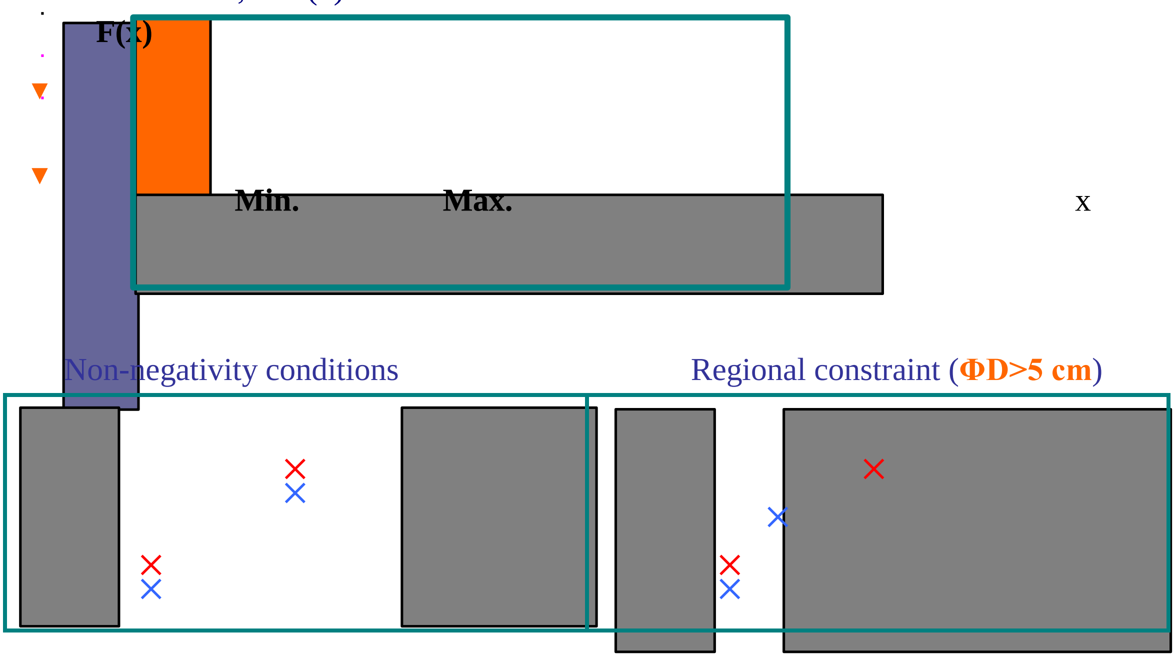
II.15 CONSTRAINED OPTIMIZATION (Regional constraints)

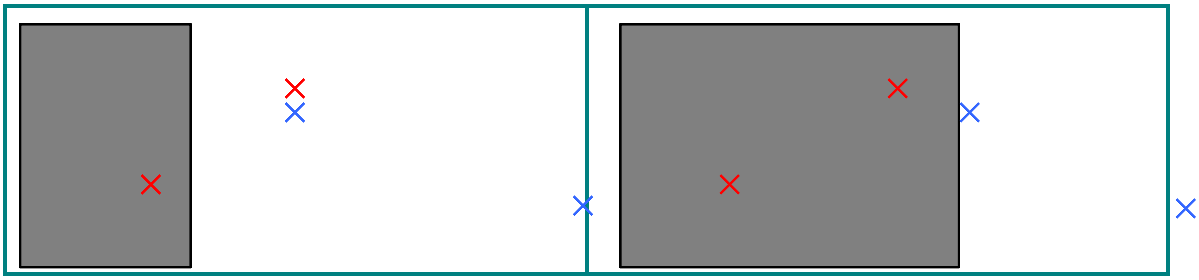
Max./Min $F = F(x_1, x_2, x_3, \ldots x_n)$

s.t.: $g_i(x_1, x_2, x_3, \ldots x_n)=0$ $i=1\ldots m$ and $\mathbf{m=0}$

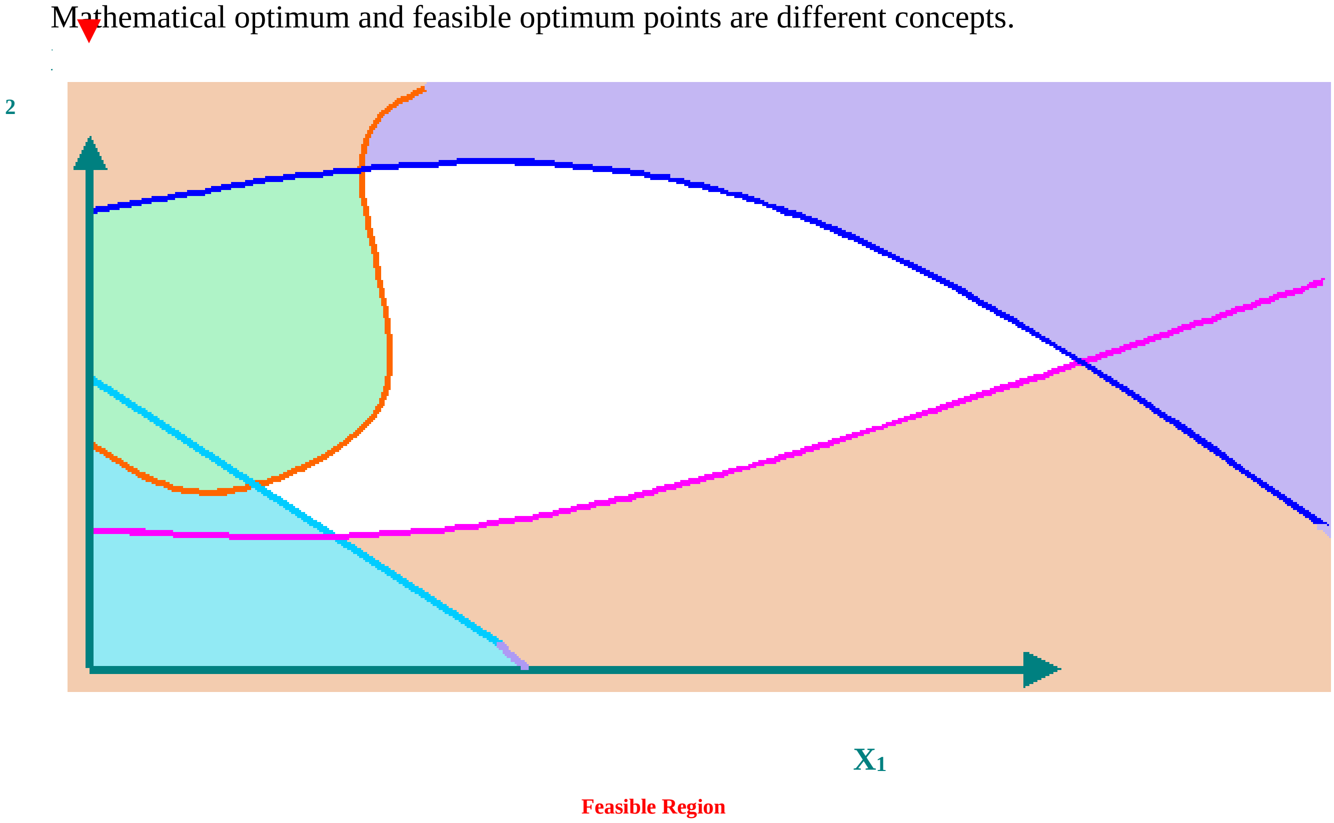
$h_j(x_1, x_2, x_3, \ldots x_n) <> 0$ $j=1\ldots p$ and $\mathbf{p>0}$

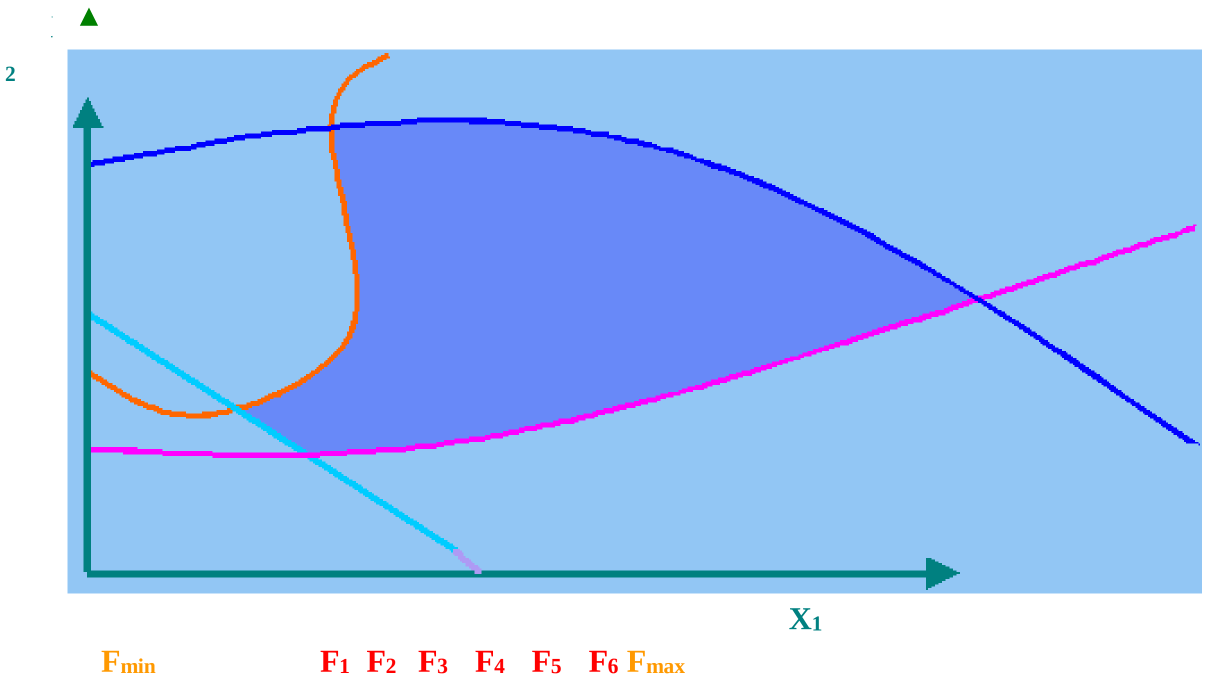
Consider n=1, F=F(x)





Mathematical optimum and feasible optimum points are different concepts.





$F_{min}<F_1<F_2<F_3<F_4<F_5<F_6<F_{max}$

Any Machine Element

4- Functional requirement parameters

5- Desirable/undesirable effects

6- Uniqueness

Design Parameters

5- Functional requirement parameters

6- Desirable/Undesirable effect parameters

7- Geometrical parameters

8- Material parameters

Procedure

Step 1- Draw a free hand sketch. Select independent design parameters to define the artifact uniquely.

Step 2- Decide on the most significant criterion and write $F=F(x_1, x_2, ...,x_n)$

Step 3- Write all related equations $g(x_1, x_2, ...,x_n)=0$.

Step 4- Write limit relations. $h(x_1, x_2, ...,x_n)><0$.

Step 5- Eliminate common parameters between g and h.

Rules: Eliminate unlimited parameters

Do not eliminate material parameters

Step 6- Draw rough sketches for F vs x_i for all x_i except material parameters.

Step 7- Apply mathematical optimization techniques to find optimum solution.

Step 8- Determine material selection factor and select the optimum material.

Step 9- Determine the optimum values of the eliminated parameters.

List of significant criteria

C) List the criteria in the order of significance

F_x $\rightarrow$ Objective Function / Primary Design Criterion

F_y

F_u

$\rightarrow$ $g(x_1, x_2, ...,x_n)=0$ or $h(x_1, x_2, ...,x_n)><0$.

F_z

PART III
CASE STUDIES and DESIGN ENGINEERING APPLICATIONS

Case Study 1

Case Study 2

Case Study 3

Case Study 4

Case Study 5

CASE STUDY 1

Solve for the optimum values as formulated here. If some of the parameters are not determined, drop F_x, move F_v up and re-solve the optimization problem. Repeat this procedure until all of the design parameters are determined.

D) Define an equivalent criterion function.

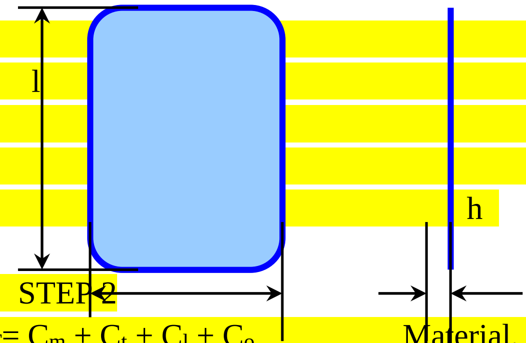
$F=\psi_1 F_1 + \psi_2 F_2 + \psi_3 F_3 + \ldots \psi_k F_k$

Where $\psi_1, \psi_2, \psi_3, \ldots \psi_k$ are weighing factors.

Limits

- Loose limits
- Rigid limits

STEP 1



V=2000 cm^3

h should not exceed 2 cm

thickness (t) material should be at least 3 mm

STEP 2

$C_T = C_m + C_t + C_l + C_o$ Material, Tooling, Labor, Overhead costs.

$C_m = c_m \rho V_m$ and $V_m = wlt + 2wht + 2lht$

Therefore

$C_m = c_m \rho t\ (wl + 2wh + 2lh)$ $\Leftrightarrow$ $F=F(x_1, x_2, x_3, x_n)$

STEP 3

$V = wlh = 2000\ cm^3$ $\Leftrightarrow$ $g(x_1, x_2, x_3, x_n)=0$

STEP 4

$h \le 2$ cm

$t \ge 3$ cm $\Leftrightarrow$ $h_j(x_1, x_2, x_3, x_n) \ge 0$

$C_T\ \alpha\ C_m$ other cost factors are fixed.

Min.	$C_m = c_m \rho t\ (wl + 2wh + 2lh)$	$F=F(x_1, x_2, x_3, x_n)$
s.t .:	$wlh - 2000 = 0$	$g(x_1, x_2, x_3, x_n)=0$
	$h - 2 \le 0$	$h_1(x_1, x_2, x_3, x_n) \ge 0$
	$t - 0.3 \ge 0$	$H_2(x_1, x_2, x_3, x_n) \ge 0$

STEP 5

$w=V / lh$ Eliminate w in $C_m = c_m \rho t\ (wl + 2wh + 2lh)$ $x_r=r(x_1, \ldots, x_n)$

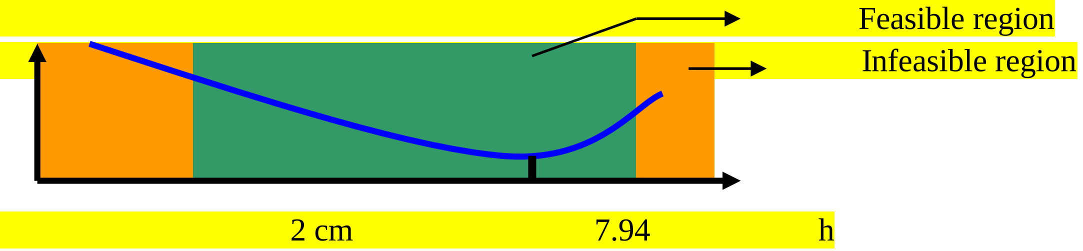
$C_m = c_m \rho t\ (2000/h + 4000/l + 2lh)$

$c_m\rho$: Material parameters group (Material Selection Factor)
t, h, l are independent geometrical parameters

STEP 6

t C_m therefore for minimum C_m, t should be minimum.

T_{opt}=3 mm



STEP 7

C_m= K (V/h + 2V/l + 2lh) (V=2000 is fixed)

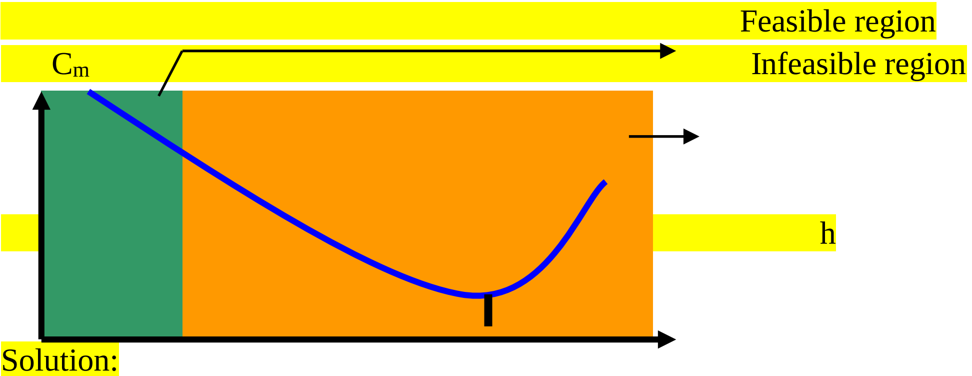
$\delta C_m/\delta h$= K (- V/h^2 + 2l)=0

$\delta C_m/\delta l$= K (- V/l^2 + 2h)=0

Solution:

$l_{opt}=(2V)^{1/3}$ = 15.87 cm

$h_{opt}=(V/4)^{1/3}$ = 7.94 cmIF



Solution:

h_{opt}= 2 cm

l_{opt}= V / h = 31.6 cm

w_{opt}= V / h = 31.6 cm

Further suppose that w < 20 cm

w_{opt} = 20 cm

l_{opt} = 50 cm

STEP 8

Material Selection factor: MSF = $c_m\rho$

Minimum $c_m\rho$

$c_m\rho$	Material
___	A
___	B
___	C
……	….

STEP 9

Compute w_{opt}Ł = $c_m \rho t$ (wl + 2wh + 2lh) + λ (wlh - 2000)

solving d Ł = 0 should give thesame result.

CASE STUDY 2

$$(PE)_T = (PE)_F + (PE)_W$$

$$(PE)_F = F\Delta_F \quad \textbf{since} \quad \Delta_F = \frac{FL^3}{3EI}$$

$$(PE)_F = \frac{F^2L^3}{3EI}$$

$$(PE)_W = \int_0^L \Delta_W w dx = \int_0^L \frac{w}{24EI}(x^4 - 4L^4x + 3L) w dx$$

$$(PE)_W = \frac{w^2L^5}{20EI}$$

$$(PE)_T = \frac{1}{EI}\left(\frac{1}{3}F^2L^3 + \frac{1}{20}w^2L^5\right)$$

This equation must be maximized.

Maximize $$(PE)_T = \frac{1}{EI}\left(\frac{1}{3}F^2L^3 + \frac{1}{20}w^2L^5\right)$$

S. t.: $$\sigma_{max} - \frac{Lh}{4I}(2F + wL) = 0$$

$$w - A\gamma = 0$$

$$W - wL = 0$$

$$\Delta_W - \frac{wL^4}{8EI} = 0$$

$$\Delta_F - \frac{FL^3}{3EI} = 0$$

$$\Delta_T - \Delta_F - \Delta_W = 0$$

$$\sigma_{all} - \frac{Y_m}{N} = 0$$

$$I - \frac{bh^3}{12} = 0 \qquad\qquad I - \frac{\pi d^4}{64} = 0$$

$$A - bh = 0 \qquad\qquad A - \frac{\pi d^2}{4} = 0$$

$$b - 5h = 0 \qquad\qquad d - 5 > 0$$

$$d - 200 < 0$$

$$\sigma_{max} - \sigma_{all} < 0$$

$$W - 80 < 0$$

$$\Delta_W - 17.5 < 0$$

$$\Delta_T - 100 < 0$$

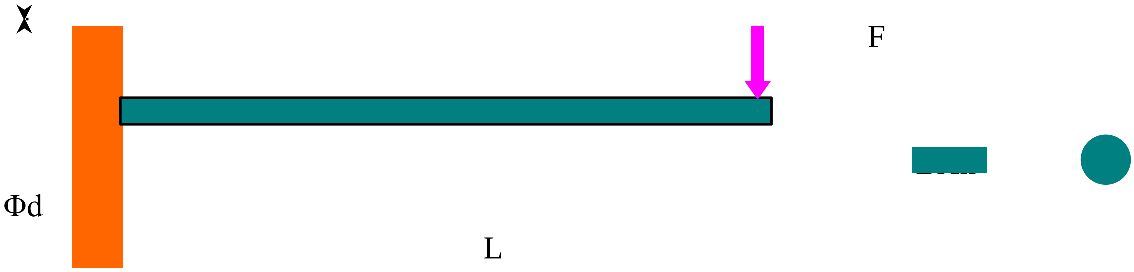
$$N - 4 > 0$$

Design parameters: $F, w, w, L, b, h, d, A, I, \Delta_F, \Delta_W, \Delta_T, \gamma, \sigma_{all}, \sigma_{max}, N$

N=16

M=10 or 9

CASE STUDY 2 EXAMPLE 2



$$(PE)_T = (PE)_F + (PE)_W$$

$$(PE)_F = F\Delta_F$$ **since** $$\Delta_F = \frac{FL^3}{3EI}$$

$$(PE)_F = \frac{F^2L^3}{3EI}$$

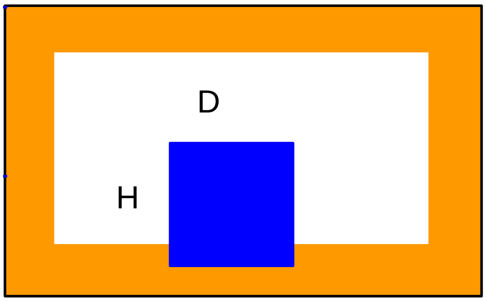
$$(PE)_W = \int_0^L \Delta_W w dx = \int_0^L \frac{w}{24EI}(x^4 - 4L^4x + 3L) w dx$$

$$(PE)_W = \frac{w^2L^5}{20EI}$$

$$(PE)_T = \frac{1}{EI}\left(\frac{1}{3}F^2L^3 + \frac{1}{20}w^2L^5\right)$$

This equation has to be maximized.

CASE STUDY 3



Insulator

Solar collector as heat source

Convection and Radiation

$q = q_c + q_r$

Max. $\quad q = hA_s(T_c - T_a) + \Sigma\sigma A_s\left(T_c^4 - T_a^4\right)$

s.t.: $\quad Q - cW\left(T_c - T_a\right) = 0$

$$W - \frac{\pi D^2}{4} Hw = 0$$

$$A - \left(\frac{\pi D^2}{4} + \pi DH\right) = 0$$

$$R - H/D = 0$$

$$0.2 \le R \le 5.0$$

$n = 10$

Geometrical Parameters : R, D, H, A_s, W

Material Parameters : W, ε, h, c

Functional Reg. Parameters: Q_s

Q is known, R is limited.

Eliminate unlimited parameters, D, H, A_s, W

$$q = \underbrace{\left[\frac{1}{4R^{2/3}} + R^{1/3}\right]}_{Geo.} \underbrace{\left[\frac{4\pi^{1/2} Q_s}{T_c - T_a}\right]^{2/3}}_{Const} \underbrace{\left[\frac{h(T_c - T_a) + \Sigma G(T_c^4 - T_a^4)}{(WC)^{2/3}}\right]}_{mtl.}$$

$$Ropt = 5.0$$

$$MSF = \frac{880 + 1390\varepsilon}{(wc)^{2/3}}$$

$$R_0 = 5.0$$

Maximize $$(PE)_T = \frac{1}{EI}\left(\frac{1}{3}F^2L^3 + \frac{1}{20}w^2L^5\right)$$

S. t.:

$$\sigma_{max} - \frac{Lh}{4I}(2F + wL) = 0$$

$$w - A\gamma = 0$$

$$W - wL = 0$$

$$\Delta_W - \frac{wL^4}{8EI} = 0$$

$$\Delta_F - \frac{FL^3}{3EI} = 0$$

$$\Delta_T - \Delta_F - \Delta_W = 0$$

$$\sigma_{all} - \frac{Y_m}{N} = 0$$

$$I - \frac{bh^3}{12} = 0$$

$$A - bh = 0$$

$$b - 5h = 0$$

$$I - \frac{\pi d^4}{64} = 0$$

$$A - \frac{\pi d^2}{4} = 0$$

$$d - 5 > 0$$

$$d - 200 < 0$$

$$\sigma_{max} - \sigma_{all} < 0$$

$$W - 80 < 0$$

$$\Delta_W - 17.5 < 0$$

$$\Delta_T - 100 < 0$$

$$N - 4 > 0$$

Design parameters: $F, w, w, L, b, h, d, A, I, \Delta_F, \Delta_W, \Delta_T, \gamma, \sigma_{all}, \sigma_{max}, N$

N=16

M=10 or 9

P=5 or 7

Q.1 Is it possible to develop a mathematical model for the physical system?

T.1 Apply Logical Optimization Techniques

Q.2 Are the constraints and criteria function linear?

T.2 Apply Linear Programming Techniques

Q.3 Is there any regional constraints?

PART IV
DESIGN ENGINEERING FOCUSED CASE STUDIES & PROBLEMS for FURTHER STUDIES